

The previous relationship, the definition of  $p[k]$ , and Lemma 6.2 yield

$$\max_{m,n} \|v_{m,n}[k]\|_2 \leq r \sqrt[p_{\Delta\eta}]{(\eta + \Delta\eta)^{k/N}}. \quad (46)$$

$U[k]$  is a parallelogram with sides  $v_{i,j}[k]$ . The triangle inequality implies

$$d_{\max}(U[k]) \leq \sum_{i=1}^P \sum_{j=1}^{n_i} \|v_{i,j}[k]\|_2 \leq N \max_{m,n} \|v_{m,n}[k]\|_2.$$

Substituting (46) into the aforementioned bound on  $d_{\max}(U[k])$  yields  $d_{\max}(U[k]) \leq \lambda_0(\eta + \Delta\eta)^{k/N}$  where  $\lambda_0 = Nr \sqrt[p_{\Delta\eta}]{}{}$ . By choosing

$$\lambda_{\Delta\eta} = \max\left(\max_{m \in [1, K_0-1]} (d_{\max}(U[m]) (\eta + \Delta\eta)^{-m/N}), \lambda_0\right)$$

we can guarantee that (17) holds for all  $k$ .  $\diamond$

#### REFERENCES

- [1] S. Haykin and S. Moher, *Modern Wireless Communications*. Upper Saddle River, NJ: Prentice-Hall, 2003.
- [2] D. Delchamps, "Stabilizing a linear system with quantized state feedback," *IEEE Trans. Autom. Control*, vol. 35, no. 8, pp. 916–924, Aug. 1990.
- [3] W. Wong and R. Brockett, "Systems with finite communication bandwidth constraints—Part II: Stabilization with limited information feedback," *IEEE Trans. Autom. Control*, vol. 44, no. 5, pp. 1049–1053, May 1999.
- [4] J. Baillieul, "Feedback designs in information-based control," in *Proc. Workshop Stochastic Theory Control*, B. Pasik-Duncan, Ed., 2002, pp. 35–57.
- [5] F. Fagnani and S. Zampieri, "Stability analysis and synthesis for scalar linear systems with a quantized feedback," *IEEE Trans. Autom. Control*, vol. 48, no. 9, pp. 1569–1584, Sep. 2003.
- [6] N. Elia and S. Mitter, "Stabilization of linear systems with limited information," *IEEE Trans. Autom. Control*, vol. 46, no. 9, pp. 1384–1400, Sep. 2001.
- [7] M. Fu, "Robust stabilization of linear uncertain systems via quantized feedback," in *Proc. IEEE Conf. Decision and Control*, Maui, HI, 2003, pp. 199–203.
- [8] R. Brockett and D. Liberzon, "Quantized feedback stabilization of linear systems," *IEEE Trans. Autom. Control*, vol. 45, pp. 1279–1289, Jul. 2000.
- [9] D. Liberzon, "On stabilization of linear systems with limited information," *IEEE Trans. Autom. Control*, vol. 48, no. 2, pp. 304–307, Feb. 2003.
- [10] S. Tatikonda and S. Mitter, "Control under communication constraints," *IEEE Trans. Autom. Control*, vol. 49, no. 7, pp. 1056–1068, Jul. 2004.
- [11] S. Tatikonda, "Control under communication constraints," Ph.D. dissertation, Mass. Inst. Technol., Cambridge, MA, 2000.
- [12] G. Nair and R. Evans, "Exponential stabilisability of finite dimensional linear systems with limited data rates," *Automatica*, vol. 39, pp. 585–593, 2003.
- [13] J. Hespanha, A. Ortega, and L. Vasudevan, "Toward the control of linear systems with minimum bit-rate. presented at *Proc. Int. Symp. Mathematical Theory of Networks and Systems*. [Online] <http://www.nd.edu/~mtns/talksalph.htm>

## Global Low-Rank Enhancement of Decentralized Control for Large-Scale Systems

A. I. Zečević and D. D. Šiljak

**Abstract**—This note proposes a new control strategy which is computationally attractive for systems of large dimensions. The main idea is to supplement decentralized feedback with a global additive term, which is computed as a product of two low-rank matrices. This feature is of critical importance for systems that cannot be adequately stabilized using standard decentralized control. The low-rank matrices can be efficiently obtained using linear matrix inequalities, and the resulting control is suitable for implementation in a multiprocessor environment. Simulations on a platoon of vehicles demonstrate that such a control can significantly improve the robustness of the closed-loop system with respect to uncertain nonlinearities.

**Index Terms**—Decentralized control, large-scale systems, linear matrix inequalities (LMIs), nonlinear systems, robust control.

#### I. INTRODUCTION

Decentralized control has long been recognized as a suitable strategy for stabilizing large-scale systems. Over the past few decades, a vast body of literature has become available on this subject, including a number of comprehensive surveys (see, e.g., [1]–[4] and the references therein). One of the most appealing features of decentralized feedback has been the fact that it requires only locally available states. Since this type of information structure constraint is common to many practical large-scale systems, it is not surprising that decentralized control has found a wide variety of applications, ranging from power systems and aerospace design to ecological models [1]. In recent years, the computational advantages of this approach have also attracted considerable attention, particularly in the context of parallel processing.

In designing decentralized control, it is common practice to view the overall system as an interconnection of  $N$  smaller subsystems

$$S_i : \dot{x}_i = A_{ii}x_i + B_i u_i + \sum_{j=1}^N A_{ij}x_j + h_i(x) \quad (1)$$

where  $x_i \in R^{n_i}$  are the local states,  $u_i \in R^{m_i}$  are the inputs, and  $h_i : R^n \rightarrow R^{n_i}$  are the nonlinear interconnections. Defining  $A_D = \text{diag}\{A_{11}, \dots, A_{NN}\}$ ,  $A_C = (A_{ij})$ ,  $B_D = \text{diag}\{B_1, \dots, B_N\}$ , and  $h(x) = [h_1^T(x), \dots, h_N^T(x)]^T$ , the model in (1) can be expressed in a more compact form as

$$S : \dot{x} = A_D x + B_D u + A_C x + h(x). \quad (2)$$

Given the structure of (2), it is natural to look for a feedback control law

$$u = K_D x \quad (3)$$

where  $K_D = \text{diag}\{K_{11}, \dots, K_{NN}\}$  is a *block-diagonal* gain matrix. The effectiveness of this approach hinges on our ability to efficiently compute a matrix  $K_D$  that stabilizes the closed-loop system

$$S_F : \dot{x} = (A_D + B_D K_D)x + A_C x + h(x). \quad (4)$$

A powerful technique for obtaining such a feedback law is based on linear matrix inequalities (LMIs) [5]–[8] and the mathematical framework proposed in [9] and [10]. In this approach, the computation of

Manuscript received May 10, 2004; revised December 16, 2004. Recommended by Associate Editor A. Bemporad. This work was supported by the National Science Foundation under Grant ECS-0099469.

The authors are with the Department of Electrical Engineering, Santa Clara University, Santa Clara, CA 95053 USA (e-mail: azecevic@scu.edu).

Digital Object Identifier 10.1109/TAC.2005.847054

$K_D$  is formulated as a convex optimization problem, which is designed to maximize the system robustness with respect to uncertainties. This method was found to be computationally attractive, and was successfully applied in the design of turbine/governor and exciter control in electric power systems [11], [12].

It is important to recognize that LMI-based decentralized control design still faces several significant challenges. The following three are of particular importance.

- i) There are certain classes of systems that are controllable, but cannot be stabilized by decentralized feedback. Systems of this type have been studied extensively in the context of decentralized (and structurally) fixed modes [13]–[17].
- ii) Decentralized control designs based on linear matrix inequalities generally require a block-diagonal Lyapunov function. Such a constraint is often restrictive, and can significantly degrade the robustness of the closed-loop system. In some cases, it can even lead to infeasibility of the optimization.
- iii) Even with an unconstrained Lyapunov function, a decentralized feedback law need not guarantee a sufficient degree of robustness with respect to uncertainties.

The main objective of this paper will be to propose a design strategy that can address the deficiencies pointed out before. Our basic idea will be to supplement decentralized feedback laws with a *low-rank centralized correction*, which can be obtained by a modification of the LMI optimization proposed in [9]. It will be established that such a correction is easy to compute for systems of large dimensions, and can be implemented efficiently in a multiprocessor environment. The effectiveness of the proposed strategy will be demonstrated on a platoon of moving vehicles.

## II. CONTROL DESIGN IN THE LMI FRAMEWORK

Let us consider a nonlinear system described by the differential equations

$$\dot{x} = Ax + h(x) + Bu \tag{5}$$

where  $x \in R^n$  is the state of the system and  $u \in R^m$  is the input vector.  $A$  and  $B$  are constant  $n \times n$  and  $n \times m$  matrices (with no specific assumptions regarding their structure), and  $h : R^n \rightarrow R^n$  represents a piecewise-continuous nonlinear function satisfying  $h(0) = 0$ . It is assumed that the term  $h(x)$  can be bounded by a quadratic inequality

$$h^T(x)h(x) \leq \alpha^2 x^T H^T H x \tag{6}$$

where  $H$  is a constant matrix, and  $\alpha > 0$  is a scalar parameter. This parameter can be viewed as a measure of robustness with respect to uncertainties in the system.

Given a linear feedback control law

$$u = Kx, \tag{7}$$

the global asymptotic stability of the closed-loop system can be established using a Lyapunov function

$$V(x) = x^T P x \tag{8}$$

where  $P$  is a symmetric positive-definite matrix (denoted  $P > 0$ ). Sufficient conditions for stability are well known, and can be expressed as a pair of inequalities

$$\begin{aligned} & x^T P x > 0 \\ & \begin{bmatrix} x \\ h \end{bmatrix}^T \begin{bmatrix} (A + BK)^T P + P(A + BK) & P \\ P & 0 \end{bmatrix} \begin{bmatrix} x \\ h \end{bmatrix} < 0 \end{aligned} \tag{9}$$

which must hold for every  $x \neq 0$ . Defining  $Y = \tau P^{-1}$  (where  $\tau$  is a positive scalar),  $L = KY$  and  $\gamma = 1/\alpha^2$ , the control design can now be formulated as an LMI problem in  $\gamma, \kappa_Y, \kappa_L, Y$ , and  $L$  [9].

*Problem 1:* Minimize  $a_1 \gamma + a_2 \kappa_Y + a_3 \kappa_L$  subject to

$$Y > 0 \tag{10}$$

$$\begin{bmatrix} AY + YA^T + BL + L^T B^T & I & YH^T \\ I & -I & 0 \\ HY & 0 & -\gamma I \end{bmatrix} < 0 \tag{11}$$

$$\gamma - \frac{1}{\bar{\alpha}^2} < 0 \tag{12}$$

and

$$\begin{bmatrix} -\kappa_L I & L^T \\ L & -I \end{bmatrix} < 0; \quad \begin{bmatrix} Y & I \\ I & \kappa_Y I \end{bmatrix} > 0 \tag{13}$$

Several comments need to be made regarding this design procedure.

*Remark 1:* The control design is formulated as a convex optimization problem, which ensures computational efficiency. The gain matrix is obtained directly as  $K = LY^{-1}$ , with no need for trial and error procedures.

*Remark 2:* The norm of the gain matrix is implicitly constrained by inequalities (13), which imply that  $\|K\| \leq \sqrt{\kappa_L \kappa_Y}$ . This is necessary in order to prevent unacceptably high gains that an unconstrained optimization may otherwise produce [9], [10].

*Remark 3:* If the LMI optimization is feasible, the resulting gain matrix stabilizes the closed-loop system for *all* nonlinearities satisfying (6). Condition (12) additionally secures that  $\alpha$  is greater than some desired value  $\bar{\alpha}$ .

*Remark 4:* The obtained controllers are linear, so their implementation is straightforward and cost effective.

A closer inspection of the optimization described in (10)–(13) clearly indicates that this is not a suitable framework for large-scale applications. Indeed, observing that the overall number of LMI variables associated with matrices  $Y$  and  $L$  is

$$\eta(Y, L) = \frac{n(n+1)}{2} + mn \tag{14}$$

it follows that the computational effort becomes prohibitively large as the system size increases. For systems of the form (2), a natural way to reduce the number of variables would be to look for a solution of Problem 1 in which matrices  $Y$  and  $L$  are *block-diagonal*, with blocks of sizes  $n_i \times n_i$  and  $m_i \times n_i$ , respectively. The number of LMI variables would then become

$$\eta(Y_D, L_D) = \sum_{i=1}^N \left[ \frac{n_i(n_i+1)}{2} + m_i n_i \right] \tag{15}$$

and the resulting gain matrix  $K_D = L_D Y_D^{-1}$  would correspond to a *decentralized control law*.

Although the use of block-diagonal matrices  $Y_D$  and  $L_D$  has the potential to drastically reduce the computational effort, it can also result in a lower robustness bound  $\alpha$  (compared to the one obtained using centralized control). In some cases, the LMI optimization can even become infeasible. With that in mind, we now propose a new design strategy in which the decentralized control is supplemented by a *low-rank centralized correction*. Our specific objective will be to design a feedback of the form

$$u = (K_D + WV)x \tag{16}$$

where  $W$  and  $V$  are matrices of dimension  $m \times r$  and  $r \times n$ , respectively, and  $K_D = \text{diag}\{K_{11}, \dots, K_{NN}\}$  corresponds to decentralized feedback. The sizes of these matrices are determined by the user, the

only constraint being that  $r \ll n$ . Such a constraint secures that the computational effort associated with the correction term remains minimal, and allows for easy implementation in a multiprocessor environment.

In designing a control law of the form (16), it is first necessary to establish whether or not the system can be stabilized by decentralized feedback. Two possible scenarios can arise in this context, leading to different design strategies.

#### A. Systems Where Decentralized LMI Design is Infeasible

In cases where Problem 1 is infeasible with block-diagonal matrices  $L_D$  and  $Y_D$ , we propose to look for a solution in the form

$$\begin{aligned} Y &= Y_D + UY_C U^T \\ L &= L_D + L_C U^T \end{aligned} \quad (17)$$

where

- 1)  $Y_D$  is an unknown symmetric block diagonal matrix, with blocks of dimension  $n_i \times n_i$ ;
- 2)  $L_D$  is an unknown block diagonal matrix, with blocks of dimension  $m_i \times n_i$ ;
- 3)  $U$  is a fixed  $n \times r$  matrix of full rank;
- 4)  $Y_C$  is an unknown symmetric  $r \times r$  matrix;
- 5)  $L_C$  is an unknown matrix of dimension  $m \times r$ .

For any given choice of  $U$ , Problem 1 becomes an LMI optimization in  $\gamma, \kappa_Y, \kappa_L, Y_D, Y_C, L_C$  and  $L_D$ . To see the connection between (17) and the desired feedback structure (16), we should observe that  $Y^{-1}$  can be expressed using the Sherman–Morrison formula as (e.g., [18])

$$Y^{-1} = Y_D^{-1} - S R U^T Y_D^{-1} \quad (18)$$

with

$$\begin{aligned} S &= Y_D^{-1} U Y_C \\ R &= [I + U^T S]^{-1}. \end{aligned} \quad (19)$$

Since  $K = LY^{-1}$ , it is easily verified that this matrix can be represented as  $K = K_D + WV$ , where

$$K_D = L_D Y_D^{-1} \quad (20)$$

is the decentralized control term and

$$W = L_C (I - U^T S R) - L_D S R \quad (21)$$

$$V = U^T Y_D^{-1} \quad (22)$$

are matrices of dimension  $m \times r$  and  $r \times n$ , respectively.

*Remark 5:* Although  $U$  introduces an additional degree of freedom into Problem 1, it is by no means clear how to choose this matrix in an optimal manner. An obvious possibility would be to treat  $U$  as an optimization variable. We should note, however, that this results in a nonlinear problem, which is undesirable in the case of large-scale systems. An alternative approach involves the development of heuristic strategies for constructing  $U$  in a way that is conducive to the feasibility of the LMI optimization process. One such method has recently been proposed in [19], and preliminary simulations suggest that it generally produces a higher robustness bound  $\alpha$  than random choices of  $U$ .

*Remark 6:* It should be noted that  $Y = Y_D$  and  $L = L_C U^T$  is a simpler choice than (17), leading to a low-rank centralized feedback law  $u = WVx$  with  $W = L_C$  and  $V = U^T Y_D^{-1}$ . Despite the simplicity, however, it is preferable to include terms  $L_D$  and  $Y_C$  in the

optimization, since this increases the number of LMI variables and generally leads to better results in terms of robustness.

The following example illustrates how the proposed centralized correction can be used to stabilize a system with a pair of structurally fixed modes under decentralized constraints.

*Example 1:* Let us consider the lower block-triangular system

$$\dot{x} = Ax + Bu \quad (23)$$

with

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 3 & -1 & 2 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{bmatrix} \quad (24)$$

and

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \quad (25)$$

Although the pair  $(A, B)$  is controllable, it is easily verified that there are two unstable modes that are *structurally fixed* with respect to decentralized control (e.g., [3]). In order to resolve this problem, we applied a centralized correction of rank 1, using (17) with

$$U = [1 \ 1 \ 1 \ 1 \ 1 \ 1]^T. \quad (26)$$

The corresponding LMI optimization produced matrices

$$K_D = \begin{bmatrix} -0.69 & -1.53 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.31 & 1.78 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.001 \end{bmatrix} \quad (27)$$

$$W = \begin{bmatrix} -2.76 \\ -0.77 \\ 0.0003 \end{bmatrix} \quad (28)$$

and

$$V = [0.98 \ 2.02 \ 1.85 \ 1.85 \ 0 \ 2.76] \quad (29)$$

and the closed-loop system

$$\dot{x} = (A + BK_D + BWV)x \quad (30)$$

was found to be stable, with eigenvalues  $-0.86 \pm j2.82$ ,  $-1.59 \pm j0.38$ ,  $-1$ , and  $-9.31$ .

#### B. Systems Where Decentralized LMI Design is Feasible

When Problem 1 is feasible with block-diagonal matrices  $L_D$  and  $Y_D$ , it makes sense to perform the design in two separate stages. Such a strategy ensures that the system remains stable in the event of a communication failure. The procedure can be described as follows.

- STEP 1) Solve Problem 1 with block-diagonal matrices  $L_D$  and  $Y_D$ . This leads to a decentralized control law  $u = K_D x$ , where  $K_D = L_D Y_D^{-1}$ . Note that the closed-loop system is guaranteed to be stable at this point, but

its robustness with respect to uncertainties may still be inadequate.

STEP 2) Apply the low-rank centralized correction to the closed-loop system  $\tilde{A} = A + B_D K_D$ , with

$$\begin{aligned} Y &= Y_D + U Y_C U^T \\ L &= L_C U^T. \end{aligned} \quad (31)$$

In this case, the supplemental control has the form  $u = W V x$ , where  $W = L_C(I - U^T S R)$  and  $V = U^T Y_D^{-1}$ .

*Remark 7:* As noted earlier, it is possible to simplify the procedure by choosing  $Y = Y_D$ . However, including the term  $U Y_C U^T$  is desirable in practice, since it produces a Lyapunov function that is *not* block-diagonal. The benefits of using structurally unconstrained Lyapunov functions have been recognized by a number of authors (e.g., [20] and [21]).

### C. Implementation Issues

In evaluating the practical advantages of the proposed approach, it is important to recognize that the number of LMI variables associated with matrices  $Y$  and  $L$  in (17) is

$$\eta(Y, L) = \frac{r(r+1)}{2} + mr + \sum_{i=1}^N \left[ \frac{n_i(n_i+1)}{2} + m_i n_i \right]. \quad (32)$$

This is similar to the decentralized case in (15), the only difference being the variables corresponding to matrices  $Y_C$  and  $L_C$ . Given that  $r \ll n$ , it follows that the computational effort required for the proposed design remains modest even when  $n$  is large. The implementation of such a control in a multiprocessor environment is also quite straightforward. Indeed, if matrices  $W$  and  $V$  are partitioned as

$$W = [W_1^T, \dots, W_N^T]^T \quad V = [V_1, \dots, V_N] \quad (33)$$

the corresponding control scheme for processor  $i$  has the form shown in Fig. 1.

In this scheme, processor  $i$  performs multiplications involving matrices  $W_i$ ,  $V_i$ , and  $K_{ii}$ , which are of dimension  $m_i \times r$ ,  $r \times n_i$ , and  $n_i \times n_i$ , respectively. This strategy also requires a front end processor, whose main function is to assemble and distribute the subsystem information, and to form the  $r \times 1$  vector

$$z(t) = \sum_{j=1}^N V_j x_j(t) = V x(t). \quad (34)$$

The only communication tasks involved are single-node gather and scatter operations, which are known to result in low overhead. If necessary, the front end processor can also periodically recompute matrices  $W_i$ ,  $V_i$  and  $K_{ii}$ , in response to changes in the system configuration.

We should note in this context that when the number of subsystems is large, direct communication with a single "supervisory" processor may not be efficient, and can result in serious bottlenecks. In such cases, it is advisable to apply a hierarchical tree-type communication strategy such as the one commonly used in massively parallel architectures (e.g., [22]).

## III. APPLICATIONS TO VEHICLE CONTROL

To demonstrate the effectiveness of the proposed method, in this section we will consider the control of large platoons of vehicles. In a close

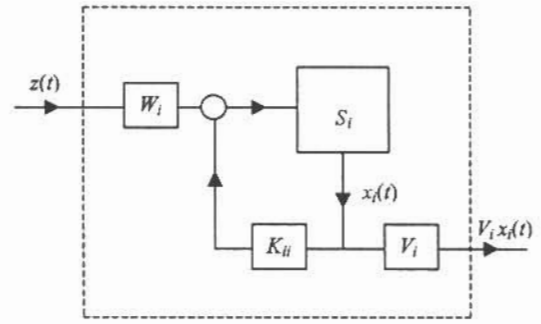


Fig. 1. Computation tasks for processor  $i$ .

formation platoon consisting of  $N$  vehicles, the  $i$ th vehicle can be represented by a nonlinear third-order model (e.g., [23]–[26])

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= f_i(v_i, a_i) + g(v_i)\eta_i. \end{aligned} \quad (35)$$

In (35),  $d_i = x_{i-1} - x_i$  represents the distance between two consecutive vehicles ( $x_{i-1}$  and  $x_i$  being their positions),  $v_i$  and  $a_i$  are the velocity and acceleration, respectively, and  $\eta_i$  is the engine input. Functions  $f_i(v_i, a_i)$  and  $g(v_i)$  are assumed to be known under normal operating conditions.

If we allow for uncertainty in  $f_i(v_i, a_i)$  due to varying external conditions, the last equation in (35) can be rewritten as

$$\dot{a}_i = f_i^0(v_i, a_i) + h_i(v_i, a_i) + g(v_i)\eta_i \quad (36)$$

where  $f_i^0(v_i, a_i)$  represents engine dynamics under nominal operating conditions and  $h_i(v_i, a_i)$  denotes the uncertain perturbation. Assuming a control law of the form

$$\eta_i = \frac{[u_i - f_i^0(v_i, a_i)]}{g(v_i)} \quad (37)$$

the dynamics of the  $i$ th vehicle can now be described as

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= h_i(v_i, a_i) + u_i \end{aligned} \quad (38)$$

which conforms to the general nonlinear model (2), with

$$h(x) = [0 \ 0 \ h_1(x) \ \dots \ 0 \ 0 \ h_N(x)]^T. \quad (39)$$

Following the scheme shown in Fig. 1, we will assume that each vehicle has its own processor, which can exchange information with a front end computer (possibly located on a satellite). Our objective in the following will be to design a control law of the form (16) that stabilizes the system for *any* perturbation  $h(x)$  such that

$$h^T(x)h(x) \leq \alpha^2 x^T x \quad (40)$$

(which is equivalent to setting  $H = I$  in Problem 1). In this process,  $\alpha$  represents a robustness bound that needs to be maximized in the course of the LMI optimization.

In our numerical experiments we considered a platoon of 50 vehicles, which is a large-scale dynamic system with 150 state variables. Unlike



TABLE I  
COMPUTATIONAL COMPLEXITY AND  $\alpha$  AS A FUNCTION OF THE CORRECTION RANK FOR VARIANT 1

Rank	$\eta(Y, L_C)$	$\alpha$
$r = 0$		0.11
$r = 1$	351	0.23
$r = 2$	403	0.25
$r = 3$	456	0.37
$r = 4$	510	0.40
$r = 5$	565	0.42
$r = 6$	621	0.44
$r = 7$	678	0.51
$r = 8$	736	0.55

TABLE II  
COMPUTATIONAL COMPLEXITY AND  $\alpha$  AS A FUNCTION OF THE CORRECTION RANK FOR VARIANT 2

Rank	$\eta(Y, L_C)$	$\alpha$
$r = 0$		0.09
$r = 1$	350	0.20
$r = 2$	400	0.21
$r = 3$	450	0.32
$r = 4$	500	0.34
$r = 5$	550	0.37
$r = 6$	600	0.38
$r = 7$	650	0.45
$r = 8$	700	0.48

Example 1, this type of system can be stabilized by decentralized control (in other words, Problem 1 is feasible for  $r = 0$  as well). With that in mind, the two-step procedure outlined in the previous section was applied to design a control law of the form (16). The matrix  $U$  was formed recursively for each value of  $r$ , using a modified Gram-Schmidt procedure.

In order to evaluate the relative importance of the structure of the Lyapunov function, we considered two variants of Step 2): Variant 1, where  $Y$  and  $L$  have the form assumed in (31), and Variant 2, where  $Y = Y_D$  and  $L = L_C U^T$ . In both cases, we monitored how  $\alpha$  increases with the rank of the correction term. The results of our simulations are summarized in Tables I and II, in which  $\eta(Y, L_C)$  represents the number of LMI variables in Step 2). Note that this number is not provided for  $r = 0$  (which corresponds to standard decentralized control), since Step 2) is not executed in that case.

It is readily observed from the tables that the system robustness can be significantly enhanced with a global low-rank correction, and that the overall computational effort increases quite moderately as  $r$  becomes larger. A comparison of the two tables also suggests that for this problem the increase in robustness is primarily due to the centralized nature of the controller. Indeed, the low-rank enhancement of the Lyapunov function improves parameter  $\alpha$  by no more than 20% in any given step.

#### IV. CONCLUSION

In this note, we developed a new method for enhancing the decentralized control of large-scale systems. The proposed approach is based on the construction of a supplemental global control that can be expressed as a product of two low-rank matrices. This product is added to the decentralized gain matrix in order to avoid stabilization problems and to improve the robustness of the closed-loop system. It was shown that such a composite control can be obtained in the framework of linear matrix inequalities, and can be efficiently implemented in a multiprocessor environment. Simulation results were provided for a large platoon of vehicles.

#### REFERENCES

- [1] D. D. Šiljak, *Large-Scale Dynamic Systems: Stability and Structure*. New York: North Holland, 1978.
- [2] M. Ikeda, "Decentralized control of large scale systems," in *Lecture Notes in Control and Information Sciences*, H. Nijmeijer and J. M. Schumacher, Eds. New York: Springer-Verlag, 1989, pp. 219–242.
- [3] D. D. Šiljak, *Decentralized Control of Complex Systems*. Cambridge, U.K.: Academic, 1991.
- [4] D. D. Šiljak and A. I. Zečević, "Large scale and decentralized systems," in *The Wiley Encyclopedia of Electrical and Electronics Engineering*, J. Webster, Ed. New York: Wiley, 1999, vol. 11, pp. 209–224.
- [5] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM, 1994.
- [6] J. C. Geromel, J. Bernussou, and P. Peres, "Decentralized control through parameter space optimization," *Automatica*, vol. 30, pp. 1565–1578, 1994.
- [7] J. C. Geromel, J. Bernussou, and M. C. de Oliveira, " $H_\infty$ —norm optimization with constrained dynamic output feedback controllers: decentralized and reliable control," *IEEE Trans. Autom. Control*, vol. 44, pp. 1449–1454, 1999.
- [8] L. El Ghaoui and S. Niculescu, Eds., *Advances in Linear Matrix Inequalities Methods in Control*. Philadelphia, PA: SIAM, 2000.
- [9] D. D. Šiljak and D. M. Stipanović, "Robust stabilization of nonlinear systems: The LMI approach," *Math. Prob. Eng.*, vol. 6, pp. 461–493, 2000.
- [10] A. I. Zečević and D. D. Šiljak, "Stabilization of nonlinear systems with moving equilibria," *IEEE Trans. Autom. Control*, vol. 48, pp. 1036–1040, 2003.
- [11] D. D. Šiljak, D. M. Stipanović, and A. I. Zečević, "Robust decentralized turbine/governor control using linear matrix inequalities," *IEEE Trans. Power Syst.*, vol. 17, pp. 715–722, 2002.
- [12] A. I. Zečević, G. Nešković, and D. D. Šiljak, "Robust decentralized exciter control with linear feedback," *IEEE Trans. Power Syst.*, vol. 19, pp. 1096–1103, 2004.
- [13] S. H. Wang and E. J. Davison, "On the stabilization of decentralized control systems," *IEEE Trans. Autom. Control*, vol. AC-18, pp. 473–478, 1973.
- [14] M. E. Sezer and D. D. Šiljak, "Structurally fixed modes," *Syst. Control Lett.*, vol. 1, pp. 60–64, 1981.
- [15] B. D. O. Anderson and D. J. Clements, "Algebraic characterization of fixed modes in decentralized control," *Automatica*, vol. 17, pp. 703–712, 1981.
- [16] E. J. Davison and Ü. Özgüner, "Characterizations of decentralized fixed modes for interconnected systems," *Automatica*, vol. 19, pp. 169–182, 1983.
- [17] T. Tanino and M. Tanaka, "Elimination of fixed modes in linear control structures," in *Proc. 12th IFAC World Congr.*, vol. 7, 1993, pp. 163–166.
- [18] G. Golub and C. van Loan, *Matrix Computations*. Baltimore, MD: Johns Hopkins Univ. Press, 1996.
- [19] A. I. Zečević and D. D. Šiljak, "An LMI-based method for designing static output feedback," Santa Clara Univ., Santa Clara, CA, Tech. Rep. ECS-0 099 469/30, 2004.
- [20] G. Zhai, M. Ikeda, and Y. Fujisaki, "Decentralized  $H_\infty$  controller design: a matrix inequality approach using a homotopy method," *Automatica*, vol. 37, pp. 565–572, 2001.
- [21] Y. Ebihara and T. Hagiwara, "Structured controller synthesis using LMI and alternating projection method," in *Proc. 42nd IEEE Conf. Decision and Control*, 2003, pp. 5632–5637.
- [22] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation*. Upper Saddle River, NJ: Prentice-Hall, 1989.
- [23] D. N. Godbole and J. Lygeros, "Longitudinal control of the lead car of the platoon," *IEEE Trans. Veh. Technol.*, vol. 43, pp. 1125–1135, 1994.
- [24] D. Swaroop, J. K. Hedrick, C. C. Chien, and P. Ioannou, "Comparison of spacing and headway control laws for automatically controlled vehicles," *Veh. Syst. Dyn.*, vol. 23, pp. 597–625, 1994.
- [25] S. E. Shladover, "Longitudinal control of automotive vehicles in close formation platoons," *J. Dyna. Syst. Meas. Control*, vol. 113, pp. 231–241, 1991.
- [26] S. S. Stanković, M. J. Stanojević, and D. D. Šiljak, "Decentralized overlapping control of a platoon of vehicles," *IEEE Trans. Control Syst. Technol.*, vol. 8, pp. 816–832, 2000.