MALLI PRACTICES & MATHEMATICAL PROFICIENCY ACROSS LANGUAGES

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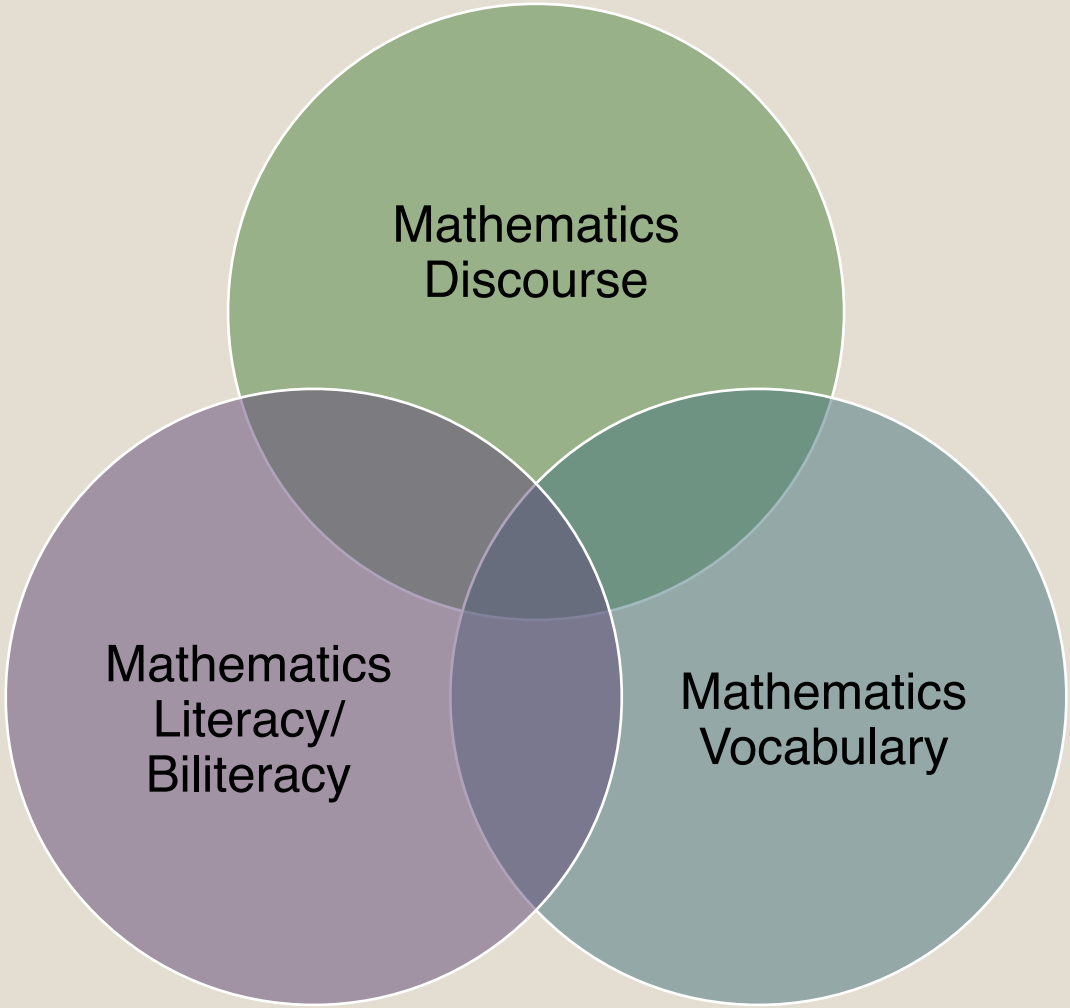
MALLI TEACHING PRACTICES



MALLI TEACHING PRACTICES

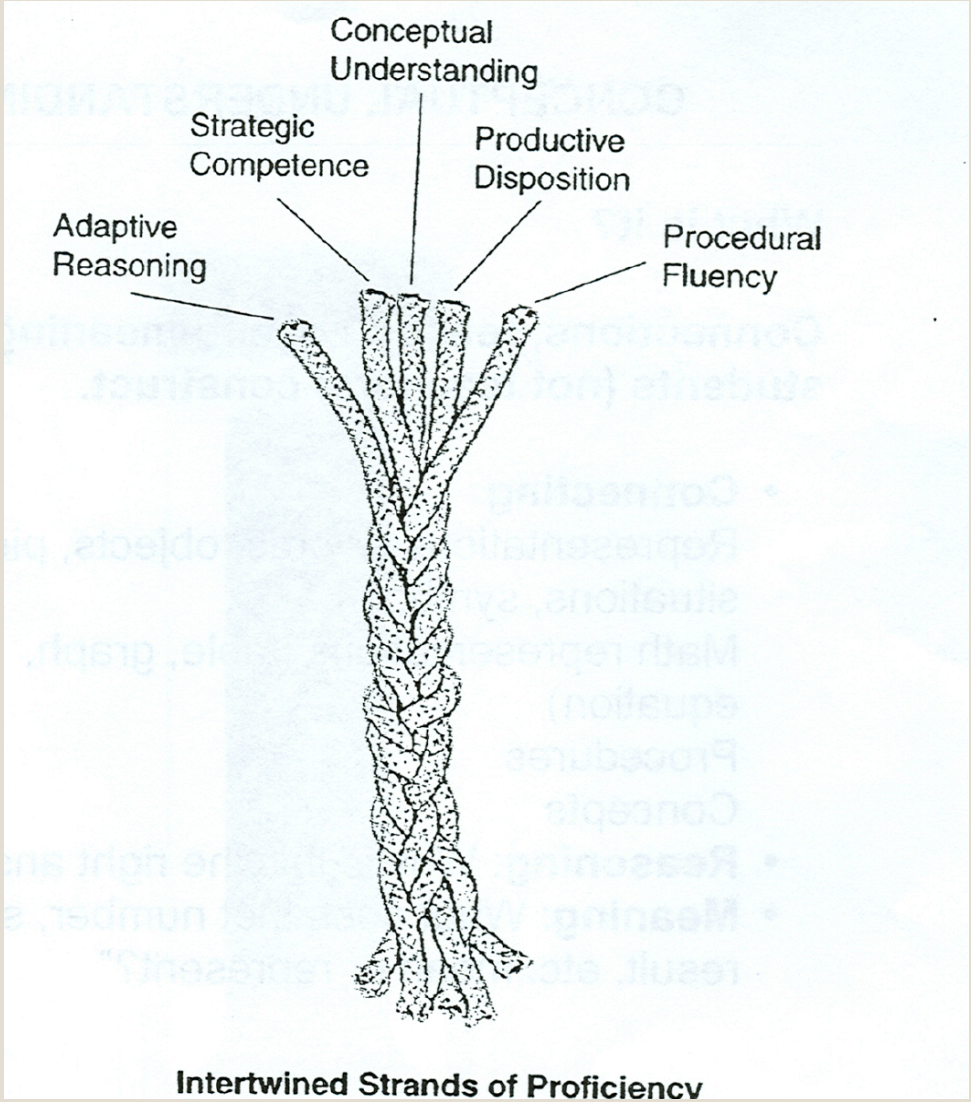
MALLI Practice	Definition	Examples of Strategies & Activities
Mathematics Discourse	Talking and acting to accomplish mathematics practices such as proving or explaining math arguments, solutions, problems, or statements	• Connect home and community funds of knowledge to school math. • Increase student interaction and collaboration. • Cultivate arguments and explanations via math talks. • Promote active listeners. • Strategies that promote math discourse and talk.

MALLI Practice	Definition	Examples of Strategies & Activities
Mathematical Literacy/Biliteracy	Attention to reading and writing in mathematics including discussions and interpretations of math texts and/or how to produce different types math texts	• math diaries • math learning logs • reflections or observations • share written responses • book of math word problems • Scaffold math textbooks
Mathematics Vocabulary	Attention to special meanings of mathematical words and reinforce specialized and precise meanings through the use of background knowledge, and specific strategies.	• Students analyze keywords in a math text: tier 1, tier 2 & tier 3 words • Root/suffix/prefix • Cognates • Collocations • Noun phrases/nominalization

MATHEMATICAL PROFICIENCY & CCSS

- The research and theory behind the common core state standards in Math.

MATHEMATICAL PROFICIENCY



Intertwined Strands of Proficiency

Adding it up: helping children learn mathematics. Kilpatrick, Swafford, & Findell (Eds.), 2001. NRC.

MATHEMATICAL PROFICIENCY STRANDS

- (1) **conceptual understanding:** comprehension of mathematical *concepts*, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean;
- (2) **procedural fluency**: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- (3) **strategic competence**: ability to formulate, *represent*, and *solve* mathematical problems using concepts and procedures appropriately;
- (4) **adaptive reasoning:** capacity for *logical thought*, reflection, *explanation*, and *justification* or to extend from something known to something not yet known; and,
- (5) **productive disposition:** habitual inclination to see mathematics as *sensible*, *useful*, and *worthwhile*, coupled with a belief in diligence and one's *own efficacy*

1. CONCEPTUAL UNDERSTANDING

- **Comprehending mathematical concepts, operations and relations—knowing what mathematical symbols, diagrams, and procedures mean.**
- Math is not isolated facts and procedures, but instead students are aware of many connections between mathematical ideas
- Conceptual understanding "... is the ability to justify … why a particular mathematical statement is true or where a mathematical rule comes from" (CCSS-M, 2010

MALLI PRACTICES AND MATHEMATICAL LANGUAGE

- Think about the language that you use language when you text? How does you language differ from when you send an email to a colleague? How does your language shift across contexts?

- How do you define mathematical language?
- Can you think of some examples of mathematical language?

MATH UNDERSTANDING & LANGUAGE "REGISTER"

- The mathematics language register includes discipline-specific words, expressions, and meanings (Halliday, 1978).
- The language of the mathematics classroom has been described as a "**register**" of technical words, expressions, and meanings that differ from those of everyday language (Schleppegrell, 2007; Secada, 1992).
- Terms have specialized meanings and also represent mathematical concepts such as "mean," "expression," "statement," "function," and "root," to name a few.
- **MALLI Practices** are trying to develop the mathematics register across languages

UNDERSTANDING: DIVIDING FRACTIONS

- Does Multiplication always yield a larger ***product*** or solution?
- Multiply: 4 by ½ and also 8 by ½
- ***Now Divide 8 by ½?***
- What does it mean in division of fractions to flip the *divisor* and multiply or "invert and multiply?"
- How can we use the ***"reciprocal"*** to understand the mathematics behind this process?
- The Reciprocal is what we multiply a value by to get 1 also "***multiplicative inverse***"

2. PROCEDURAL FLUENCY

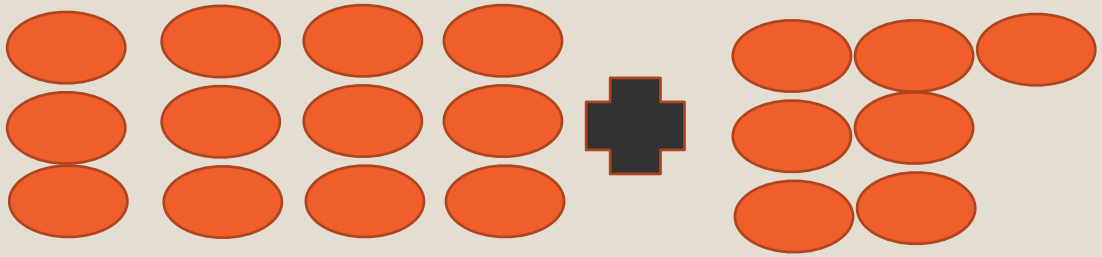
- **Carrying out procedures flexibly, accurately, efficiently, and appropriately; Involves fluency with procedures for arithmetic, precision, and using such procedures appropriately.**
- When students memorize procedures they fail to understand deeper ideas.
- Understanding makes it easier to learn skills, while learning procedures can strengthen and develop math understanding.
- Exp. 1: 62 – 48 = 26??

3. STRATEGIC COMPETENCE

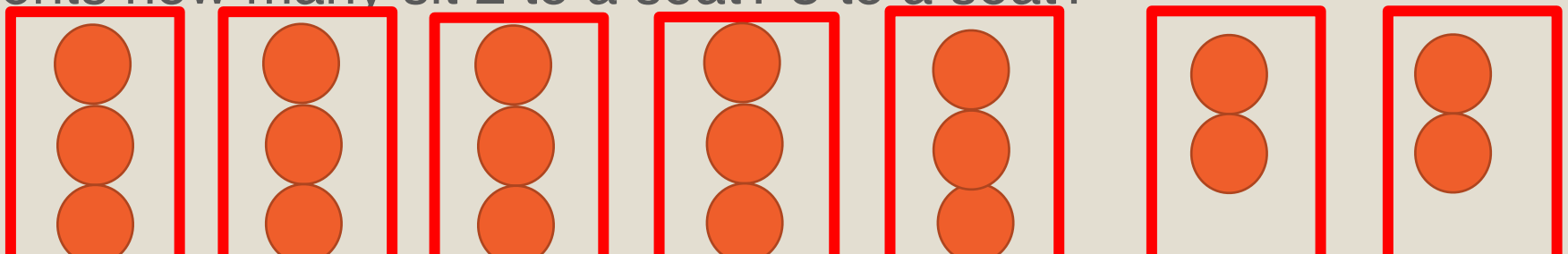
- **Ability to formulate, *represent*, and *solve* mathematical problems using concepts and procedures appropriately A concept or procedure is not useful unless students know where to use it.**
- Strategic competence refers to the ability to formulate mathematical problems, represent them, and solve them (NRC, 2001).
- To solve a problem, the student's first step is to "represent it mathematically in some fashion, whether numerically, symbolically, verbally, or graphically" (NRC, 2001, p. 124)

3. STRATEGIC COMPETENCE

- Strategic Competence key features include **problem solving** and **representations**.
- Exp1: If 12 students are on a mini bus and 7 more get on, how many Ss on the bus?



- Exp2: A minibus has 7 seats that hold 2 or 3 students. If 19 students how many sit 2 to a seat? 3 to a seat?



4. ADAPTIVE REASONING

- **Capacity for *logical thought*, reflection, *explanation*, and *justification* or to extend from something known to something not yet known.**
- Reasoning is the “glue” that holds math together.
- Thinking about the logical relationships between concepts and situations, students can see how elements of a problem fit together.
- CCSS-M describe reasoning as “making conjectures and build a logical progression of statements to explore the truth of their conjectures” and also emphasize students’ ability to “justify their conclusions, communicate them to others, and respond to the arguments of others”
- Ball and Bass (2003) have argued that reasoning is as fundamental to knowing and using mathematics as comprehension of text is to reading.

ADAPTIVE REASONING

- Best way to improve reasoning is to explain or justify a solution to others.
- Exp1: Instead of asking for the corrects answer, we ask, “Why do you think the answer is correct?”

5. PRODUCTIVE DISPOSITIONS

- Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work.
- Engaging students in mathematical activity is the key to success.
- Math proficiency goes beyond being able to understand, compute, apply, and reason.
- Students engaged with math see themselves as effective learners, doers, and users of math.
- Engaging requires frequent opportunities to make sense of math, and recognize benefits of perseverance. Also builds confidence.
- Growth Mindset (Fixed vs. Growth)
- Exp: Develop story problems using known contexts.
- Exp: Organize for small victories—to experience success

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USING STORIES TO SUPPORT MATH LEARNING

EXAMPLE 1. (KINDERGARTEN)

Context:

The teacher began the math lesson with a story about a woman who owned many cats and lived behind the school.

Example 1

1. *Ms. Arenas:* Fíjense, mis niños, fíjense que la señora, aquí atrás de la escuela, que vive aquí atrâs. …

   (Listen, my dear children, you know the woman, the woman here behind the school, that lives right here in the back [of the school].)

2. *Students:* ¡Sí! *(nodding heads, indicating that they know the woman to whom she is referring)*

3. *Ms. Arenas:* La señora tenía tres gatos.
   (The woman had three cats.)

4. *Students:* ¿¡Tres gatos!? *(Three cats?)*

Example 1

5. *Ms. Arenas:* Sí, pero luego, su hija le regaló otros tres. Y luego, todos los gatos vienen a jugar aquí a la escuela. ¿Cuántos vendrán? ¿Cuántos gatos?

   (Yes, but then, her daughter gave her another three. And then, all the cats came here to play at the school. How many came? How many cats?)

6. *Student:* Seis!

7. *Ms. Arenas:* Escuchen, la señora tenía tres gatos, y luego su hija le regaló otros tres, ¿cuántos tendrá ahora? Piensen, y ahora me van a decir.

   (Listen, the woman had three cats, and then her daughter gave her another three; how many would she have? Think about it, and then you are going to tell me.)

Strategic Competence
(Problem Solving)
Procedural Fluency

Discourse
(Revoicing)

EXAMPLE 2. (1ST GR.)

Context

- Teacher poses a problem: “Alan has 16 chocolates and Oscar has 6. How many more chocolates does Alan have?"

- A student used cubes to represent the solution to the problem.

(Musanti, Celedon-Pattichis & Marshall, 200

Example 2

1. T: Ella acá tiene 16 chocolates de Adan, a ver agárralos con una manita [She has 16 chocolates here from Adan. Take them in your hand.] (Teacher gives stack of 16 cubes to the girl.)
2. T: ¿y qué hiciste con los otros? [And then what did you do with the others?] (T is holding a stack of six cubes in her hand.)
3. S: Los, los, los . . . puse al lado [I placed them side by side.] (T gives stack of six cubes to the girl; she holds both stacks side by side.)
4. T: Los comparaste . . . [You compared them] (showing with her hands)
5. T: Ella puso los de Adan y los de Oscar [She put Adan's and Oscar's (cubes) down.] (T indicates S to show the class both stacks side by side.)
6. T: ¿Cuántos más tenía Adan? [How many more did Adan have?]
7. S: Diez. [Ten.]
8. T: ¿Cómo sábes que eran diez más? [How do you know he had 10 more?]
9. S: Porque los conté como así . . . como de arriba [Because I counted them like this . . . from the top]. (indicates with her hand the direction she counted top to bottom)

(Musanti, Celedon-Pattichis & Marshall, 2009

XAMPLE 3. (K) PT. 1

Context

- Ms. Sandra had six candies. Ms. Mary ate four. How many are left?
- After posing the problem, Ms. Elba had one student model how she solved it using her fingers and then she asked several students to go the front of the room to draw how they did the problem.

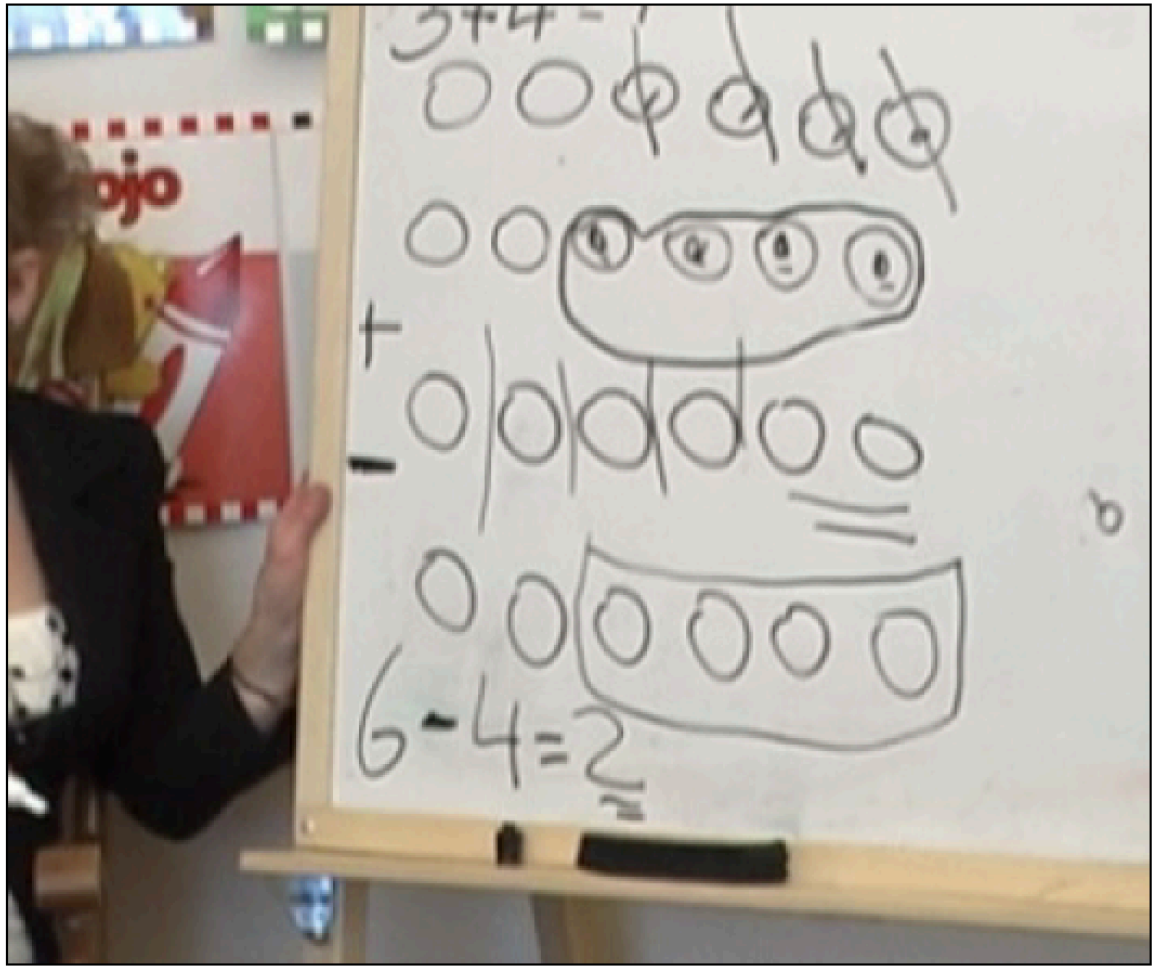

Figure 2. Students share multiple ways to represent the solution

(Musanti, & Celedon-Pattichis, 2013)

1.	Ms. Elba:	Muy bien, ¿y qué pasó? Ms. Mary qué? *(Very good, and what happened? Ms. Mary what?)*
2.	Juan:	Se los comió. *(She ate them.)*
3.	Ms. Elba:	Se comió…¿Cuántos? *(She ate…how many?)*
4.	Juan:	Cuatro. *(Four.)*
5.	Ms. Elba:	O.K., ¿cómo le hizo? *(How did she do it?)* [Juan is staring at his drawing not very sure what to do next] Cómaselos, miam, miam, miam, miam. [imitating the act of eating the candies] *(Eat them…miam, miam, miam, miam.)*

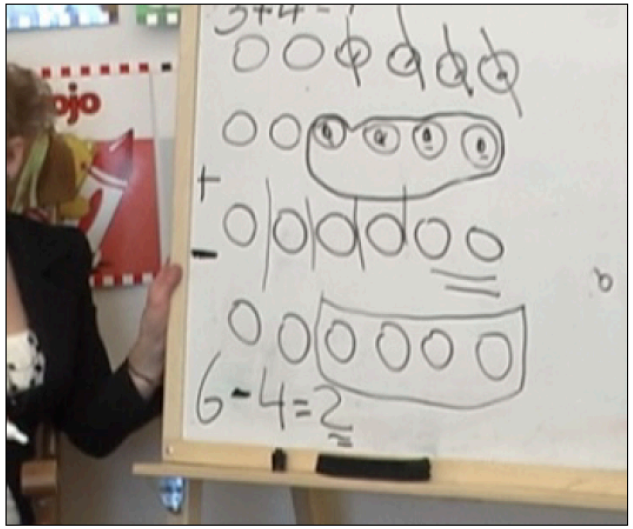

Figure 2. Students share multiple ways to represent the solution

(Musanti, & Celedon-Pattichis, 2013)

Ms. Elba then posed the following word problem,

"In my fish tank I put five fish and then Ms. Craw gave me three more. How many do I have in the fish tank?

Students began to put up one hand to illustrate five fish and three fingers to represent the set of three fish.

After a student shared and modeled his solution to the problem on the board using a drawing and adding a number sentence (5 + 3 = 8), Ms. Elba decided to challenge students and asked them to read aloud the number sentence in English for the first time.

(Musanti, & Celedon-Pattichis, 2013)

- **Students were asked to read the number sentence in English**

5+3=8

6.	Ms. Elba:	Ay! Now boys and girls Ms. Elba does not know how to read this sentence in English.
7.	Students:	Five plus three [one or two students]
8.	Ms. Elba:	Oh, oh, I didn't know, how are we reading [she is pointing to number 5].
9.	Students:	Five [Ms. Elba says five while pointing to the number.]
10.	Student:	Plus [Ms. Elba is pointing to the plus sign, reads "plus" after the student.]
11.	Student:	Three [Ms. Elba points to the three.]
	[Now, Ms. Elba is pointing to the equal sign and one student says: "same".]	
12.	Student:	Same as eight.
13.	Ms. Elba:	Five plus three same [pointing to each element in the sentence]
14.	Students:	Same like eight.

(Musanti, & Celedon-Pattichis, 2013)

- S's: "Five plus three same as eight" while others read it as "Five plus three same like eight"

15.	Ms. Elba:	O.K. Can we use another word for same? Same is O.K. Same as eight. That's O.K. I like it. But we can say five plus three. How do we say it in Spanish? [pointing to the = sign]
16.	Students:	Igual. *(Equal)*
17.	Ms. Elba:	Igual. *(Equal)* So, it's a very similar word. Five plus three equal…how many?
18.	Students:	Eight.
19.	Ms. Elba:	Eight. Whether you say same it's O.K., but we are gonna try to use the new word. Let's…everybody here…Five plus three equal…
20.	Students:	Eight.

(Musanti, & Celedon-Pattichis, 2013)

EXAMPLE. 4 (K)

- Ms. Arenas posed several problems related to the upcoming Easter holiday, including the following multistep problem:
- "Ms. Arenas had two boxes of chocolate eggs, and each box had four eggs. Then I ate two chocolate eggs. How many do I have left?"

1. Ms. A: *A ver, Dalia, tú no has pasado. Ven a explicar*. ("Let's see, Dalia, you have not come up. Come explain.") [Dalia goes up to the front of the room and takes her whiteboard.]
2. D: *Primero Ms. Arenas se comió* . . . ("First, Ms. Arenas ate . . . ")
3. Ms. A: *¿Cuántos había en cada cajita?* ("How many were there in each box?")
4. D: *Cuatro*. ("Four.")
5. Ms. A: *Okay, muy bien, y pusiste cuatro en cada cajita*. ("Okay, very good, and you put four in each box.") [She has drawn two small squares, with four circles in each square.]
6. D: (nods)
7. Ms. A: *Y luego me comí dos*. ("And then I ate two.")
8. D: *y le quedaron seis*. ("And you were left with six.")
9. Ms. A: *¿Y cómo supiste?* ("And how did you know?")
10. D: *Porque los conté, cuántas bolitas quedaron.* ("Because I counted them, how many little balls were left.")
11. Ms. A: *¿Y las otras? ¿Cómo sabes cuáles son los que comí?* ("And the others? How do you know which ones are the ones I ate?")
12. D: *Estas*. ("These.") [points to two circles that are crossed out]

EX. 4: DALIA SOLVES A MULTISTEP PROBLEM

- In this episode, Ms. Arenas used a series of probes (lines 3, 9, and 11) to elicit details of Dalia's strategy and to communicate the discursive norm that one needs to explain one's thinking.
- Additionally, she provided a frame for Dalia's explanation that was based on describing the actions in the story (Y luego me comi' dos, line 7) and explaining how she represented those actions in her drawing (Como sabes cuales son los que comi'?, line 11). We also see how Dalia
- Responded in ways that reinforced important discursive norms—by using more mathematical language (i.e., stating that six were left) and referring to visual representations (i.e., indicating that she crossed out circles in her drawing) to communicate her reasoning.

MATH PROFICIENCY & MALLI PRACTICES

Math Proficiency

Understanding Procedural Fluency Strategic Competence Reasoning Prod. Dispositions

Mathematic Representations

(Strategic Competence)

Across Languages in Dual Language Contexts

Literacy

Familiar Story Problems (Strategic Competence, Productive Dispositions Procedural Fluency)

Vocabulary

Math Language (Register) (Understanding)

Discourse

Techer moves asking for explanations (Reasoning)