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Options and structured products in behavioral portfolios

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ABSTRACT

Options and structured products have no roles in mean–variance portfolios, but they have roles in behavioral portfolios. Behavioral portfolios are composed of mental account sub-portfolios, each associated with a goal, such as retirement income or bequest. Investors optimize each mental account by finding the assets and asset allocation that maximizes the expected return of each mental account sub-portfolio subject to the condition that the probability of failing to reach a preset threshold aspiration level not exceed a preset probability. Put options are useful in ‘downside protection’ mental accounts whose goal is avoiding poverty, whereas call options are useful in ‘upside potential’ mental accounts whose goal is a shot at riches. We also explore the roles in behavioral portfolios of option collars, capital guaranteed notes, and barrier range notes.

1. Introduction

Structured products are popular among investors and many include them in their portfolios, yet that popularity is puzzling within mean–variance portfolio theory. Henderson and Pearson (2011) investigated Stock Participation Accreting Redemption Quarterly pay Securities (SPARQS), a structured product created and marketed by Morgan Stanley. One SPARQS, issued in 2004, ties cash flows to investors to the returns of the stock of National Semiconductor. Henderson and Pearson found that SPARQS are overpriced by eight percent. They wrote: “We analyze the pricing of SPARQS, the most popular listed structured equity product, and document that they are sufficiently overpriced that their expected returns are less than the riskless rate. In a standard model of portfolio selection, such securities would not rationally be purchased by an investor whose marginal utility covaries negatively with the SPARQS returns, and it is difficult to rationalize the SPARQS purchases in the context of a plausible normative model of rational investors.”

Structured products are popular in Europe even more than in the USA. Hens and Rieger (2008) noted that in 2007 alone structured products amounted to almost Seven percent of total market capitalization in Germany, and structured products amounted to more than seven percent of market capitalization in Switzerland. Hens and Rieger could not find a rationale for these notes within standard utility theory. “Thus we come to the conclusion,” they wrote, “that by and large the market for structured products, which is a huge business for banks, offers a utility gain for investors which is most likely only an illusion.” (p. 28).

We argue that while structured products are inconsistent with expected utility or mean–variance portfolio theory, many are consistent with behavioral portfolio theory even when overpriced.

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Mean–variance portfolio theory, developed by Markowitz, offers investors tools for identifying portfolios on the mean–variance efficient frontier. Portfolios on the efficient frontier have advantages over portfolios below it, but mean–variance portfolio theory tells us nothing about why investors place money in portfolios and what they plan to do with it. Behavioral portfolio theory, developed by Shefrin and Statman (2000), offers answers to these questions. Investors place money in portfolios because they have goals, whether retirement income, education expenses, or bequests. Investors divide their overall portfolios into mental account sub-portfolios, where each mental account is devoted to a goal. Behavioral portfolio theory offers efficient frontiers of risk and expected returns, one for each mental account. But risk is measured in behavioral portfolio theory by the probability of failing to reach threshold returns rather than by the standard deviation of returns. The optimal securities for each mental account in behavioral portfolio theory have return distributions that cannot be fully described by their means and standard deviations. Instead, such securities have uniform payoffs over a range of states of the world. Optimal securities for "downside protection" mental accounts resemble Treasury bills, Treasury bonds, or corporate bonds. They have uniform positive payoffs in all states of the world except the very bad ones, where defaults occur. Optimal securities for "upside potential" mental accounts resemble lottery tickets with positive payoffs in only one or few states. Stocks and options are hybrids of optimal securities. For example, a call option is a set of lottery tickets.

We find that optimal behavioral portfolios are likely to include put options, call options, and other structured products. Moreover, optimal behavioral portfolios might include both put options and call options on the same securities. Put options are often included in downside protection mental accounts. This is because put options facilitate the construction of mental accounts constrained by desires for relatively low probabilities of failing to reach relatively high threshold returns. Put options are expensive and their inclusion detracts from expected returns. This explains our finding that put options are not included in optimal mental accounts when threshold returns are set at relatively low levels.[1]

Call options are often included in upside potential mental accounts, especially when investors are willing to accept relatively high probabilities of failing to reach relatively high threshold returns. This is because call options, like lottery tickets, offer small probabilities of very high returns.

Safety collars (long puts and short calls) are used when investors set relatively high threshold returns, requiring the purchase of puts, but pay for a portion of the costs of puts by selling calls, giving up a portion of the expected returns. They may also be used to hedge background risk as analyzed in Franke et al. (1998). Aggressive collars (short puts and long calls) are used when threshold returns are set relatively low because they enable investors to take leveraged positions that enhance expected returns.

In earlier work, optimizing portfolios with derivatives has been undertaken in a standard utility framework. Using utility allows moments other than mean and variance to be accounted for in the optimization model. Milevsky and Kyrychenko (2008) adopted a static framework with exponential utility functions to analyze the use of variable-rate annuities in portfolios. Haugh and Lo (2001) applied a power utility function to show how derivatives in a static buy-and-hold framework can replicate a dynamic portfolio strategy. Liu and Pan (2003) provided dynamic asset allocation results in markets where derivatives are used to complete markets. In contrast to a utility-based approach, we maximize expected returns subject to specified probabilities of failing to reach specified threshold returns. Jaeger et al. (1995) showed the mathematical connection between the mean–variance problem and efficient shortfall.

Shefrin and Statman (2000) noted that behavioral portfolio theory begins with the attempt of Friedman and Savage (1948) to create a framework that would be consistent with the insurance–lottery puzzle, the observation that people who buy insurance policies often buy lottery tickets as well. That observation is inconsistent with expected utility theory. The mean–variance portfolio theory of Markowitz (1952) is inconsistent with the insurance–lottery puzzle. Yet two other theories introduced in 1952 are consistent with the puzzle, customary wealth theory (Markowitz, 1952) and safety-first theory (Roy, 1952). Indeed, Markowitz (1952) introduced customary wealth theory to correct deficiencies in Friedman and Savage's solution of the puzzle.

Investors are ruined when their terminal wealth falls short of a subsistence level. Investors in Roy's safety-first portfolio theory aim to minimize that probability of ruin. Whereas Roy assumed a pre-determined subsistence level, Kataoka (1963) (see Elton and Gruber, 1995) does not. The objective of Kataoka's safety-first investors is to maximize the subsistence level subject to the constraint that the probability that wealth falls below the subsistence level not exceed a predetermined level. Telser (1956) developed a model that includes a predetermined subsistence level and a predetermined probability of ruin. Telser investors are safe if the probability of falling short of a predetermined subsistence level does not exceed predetermined probability. Telser investors maximize expected wealth, subject to the condition that the probability of falling short of the predetermined subsistence level not exceed the predetermined probability.

Kahneman and Tversky (1979) built on the insights of Friedman and Savage and Markowitz in prospect theory where utility is determined not by wealth but by wealth relative to reference wealth. That reference wealth might be the current level of a person's wealth but it might also be an expected or aspired level of wealth. Indeed, Koszegi and Rabin (2006) developed a model of reference dependent preferences, where the reference level corresponds to expectations or aspirations of wealth rather than to current or status-quo levels of wealth. They noted, for example, that an employee with a $50,000 salary who expected a $60,000 salary would regard a $50,000 salary as $10,000 loss relative to his expected

[1] More detailed analysis of the amounts invested in mental accounts for different horizons is provided in De Giorgi (2011b).

salary, rather than as a zero loss relative to his $50,000 status-quo salary. Larrick et al. (2009) found, in experiments, that aspirations to meet goals increase risk-taking. A salesman who can reach his sales goal with a riskless strategy will prefer it over a risky strategy even when the risky strategy is expected to yield many more sales. Yet he will choose the risky strategy when the riskless strategy would leave him short of his goal.

Recently, Das et al. (2010) combined some features of mean–variance portfolio theory and behavioral portfolio theory in mental-accounting portfolio theory. Investors in that theory divide their portfolios into mental accounts, as in behavioral portfolio theory, but they place in them combinations of securities which can be described fully by their means and standard deviations, as in mean–variance portfolio theory. The unavailability of option-like securities in mental-accounting portfolio theory is a major drawback because thresholds in downside protection mental-accounts, such as retirement accounts, must be very low if investors want the probability of failing to reach them to be low. Adding options and structured products to the set of available securities allows benefits in the form of better combinations of expected returns and probabilities of failing to reach thresholds, even after accounting for costs and apparent overpricing.

Our analyses of capital guaranteed notes and barrier-M notes provide illustrations of these benefits. Structured products have a place in other frameworks as well. Breuer and Perst (2007) show in a cumulative prospect theory framework with hedonic framing that investors might hold discount reversible convertibles and reverse convertibles, whereas they would not do so in a power utility framework. Branger and Breuer (2008) show that retail derivatives improve portfolios for CRRA investors. Hens and Rieger (2008) extend their utility framework to prospect theory and expected utility with aspiration levels, and find that capital protected products are included in the portfolio.

Investors in the framework of Das et al. enjoy an unusual benefit. While they optimize each mental account in isolation from the others, they lose no benefits when the mental accounts are added up to form the overall portfolio. Each mental account lies on the mean–variance efficient frontier and so does the overall portfolio. This benefit is the result of mean–variance optimization, and cannot be generalized.

Shefrin and Statman presented efficient frontiers in behavioral portfolio theory, noting that the efficient frontiers of behavioral portfolio theory do not generally coincide with the efficient portfolios of mean–variance portfolio theory. For example, a lottery ticket might constitute the entire behavioral efficient frontier in an upside potential mental account for an investor who has a very high aspiration level, reflected in a very high reference point, relative to the amount she allocates to that account. That lottery ticket is most likely below the mean–variance efficient frontier. Portfolios below the mean–variance efficient frontier are inefficient by the criteria of mean–variance portfolio theory, yet efficient by the criteria of behavioral portfolio theory.

Moreover, optimization of mental accounts in isolation from others, as in behavioral portfolio theory, might lead to positions in two mental accounts that offset each other, yet the investor pays the cost of both. Similarly, investors might pay the cost of a structured security because the structure fits it in a particular mental account, whereas its components fit within different mental accounts. Zhou and Pham (2004) noted that some securities are associated in the minds of investors with promotion, or upside-potential goals and others with prevention, or downside protection goals. For example, zero-coupon bonds fit well in downside-protection mental accounts but not in upside-potential mental accounts, whereas call options on a stock market index fit well in upside-potential mental accounts but not in downside-protection mental accounts. A "homemade" structured product combining a zero-coupon bond and a call option does not fit well in either mental account as its conflicting components are exposed, but a "manufactured" structured product fits in a downside-protection account as its embedded call option is obscured.

The paper proceeds as follows. Section 2 introduces the objective function for the portfolios examined in this paper and its connection to the mean–variance paradigm. Section 3 presents an approach for optimizing portfolios with options and structured products. Using this method, we optimize portfolios with puts, calls, and collars. For robustness, we further consider the alternate risk measure of expected shortfall, i.e., we restrict the expected losses in the tail of the return distribution of the portfolio; results are qualitatively unchanged. Section 4 examines capital guaranteed notes and barrier range notes, and finds that they are attractive securities in mental accounts, despite overpricing. Section 5 presents similar analyses with non-Gaussian joint distributions. Section 6 offers concluding comments.

2. Optimizing mean–variance and behavioral portfolios

Investors in mean–variance (MV) portfolio theory attempt to identify the portfolios with the lowest variance corresponding to their preferred expected returns. Alternatively, they attempt to identify the portfolios with the highest expected returns corresponding to their preferred variance. Investors choose portfolio weights $w=[w_1,\dots,w_n]'$ for n securities where the securities have a mean return vector $\mu \in R^n$ and a return covariance matrix $\Sigma \in R^{n\times n}$.

The MV-optimization problem involves a trade-off between mean and variance subject to the full-investment constraint:

$$\max_{w} \quad w'\mu - \frac{\gamma}{2} w'\Sigma w, \quad \text{s.t. } w'\mathbf{1}=1 \tag{1}$$

where $\mathbf{1}=[1,1,\dots,1]' \in R^n$. Here the parameter γ is the risk-aversion coefficient of an investor, and it modulates the trade-off between portfolio mean and variance.

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Investors in behavioral portfolio theory choose optimal portfolios for each mental account on an efficient frontier displaying tradeoffs between expected returns and risk, measured as the probability α of failing to reach a threshold return H. Consider the following portfolio optimization problem:

$$\max_{w} \quad w'\mu, \quad \text{s.t. } \text{Prob}[r_P \le H] \le \alpha \tag{2}$$

This constraint states that the portfolio should not deliver a portfolio return r_P lower than H with a probability exceeding α. When returns are normally distributed, this constraint corresponds to the safety-first criterion of Telser (1956).

Das et al. (2010) show, under normality in mean–variance space, that there is a mapping from $\{H,\alpha\}$ in the behavioral portfolio problem to risk-aversion (γ) in the MV problem. However, the equivalence between the mean–variance portfolio problem of Eq. (1) and the behavioral portfolio problem in Eq. (2) belies a subtle difference, one that we are keenly interested in here. The standard mean–variance portfolio problem in Eq. (1) is a special case of the problem in Eq. (2) under normality. In Eq. (1) attention is restricted to only the first two moments of the portfolio's return. But the statement of the constraint in Eq. (2) is not restricted to mean–variance specified distributions. Any distribution may be imposed on joint returns. Hence this more general problem statement may be used when facing non-normal joint distributions in the optimization of portfolios. A good example of such an extension is in using multivariate T distributions, so that fatter tails in outcomes are accommodated. Later we show how copula functions may be used to apply a wide range of multivariate distributions to the problem. The portfolio payoff distribution becomes non-normal when derivatives are allowed, even when the underlying basic assets have a multivariate normal distribution.

There is a growing literature on portfolio optimization with downside risk constraints. Jaeger et al. (1995) showed the connection between mean–variance optimization and efficient shortfall. De Giorgi et al. (2011) and De Giorgi (2011a) analyzed two-fund separation in this setting and showed that the mental accounting framework is supported empirically. Boyle and Tian (2007) developed a general framework for portfolio optimization with downside or benchmark constraints. Benati and Rizzi (2007) showed that this problem is NP-hard, and offered a polynomial time algorithm to solve it for the case of a fixed number of past time periods or a fixed small number of assets.

We do not impose market completeness, nor is it necessary for the solution or the practical application. The problem we analyze is one faced by investors who solve an extended static Markowitz mean–variance framework to include derivatives, resulting in a non mean-variance setting. Investors are in a static buy-hold setting, excluding trading. The market is not complete because the number of terminal states vastly exceeds the number of available assets. Moreover, the market cannot be dynamically complete because investors do not trade. In this way, our setting is different from that of Basak and Shapiro (2001) and Boyle and Tian (2007). Further, we disallow the purchase of a continuum of securities to span all states, as that is not a practical undertaking. Our results complement those in earlier work with three additional features: (i) we analyze the roles of several options and structured products within portfolios; (ii) we develop a numerical scheme that is economical; and (iii) we show how to use copula functions in the analysis. Our approach to portfolio optimization with options and structured products is very general, extending to risk-seeking investors as well. We illustrate the application of the objective function with an example that serves as the base case throughout the paper.

2.1. Base case example

We employ an initial set of three securities, a low-risk security with an expected annual return of 5%, a medium-risk security with an expected annual return of 10% and a high-risk security with an expected annual return of 25%. The standard deviations of returns of these securities are 5%, 20% and 50% respectively. The returns of the low-risk security are uncorrelated with the returns of the other securities. The returns of the medium and high-risk securities have a correlation of 0.20.

Consider an investor who is constructing a retirement mental account. A relatively high threshold coupled with a relatively low probability of failing to reach it will result in an optimized mental account with a relatively high allocation to the low-risk security and a relatively low expected return. We begin with such an investor, one who wants the probability of failing to reach a -10% threshold return (H) not to exceed 0.05 (α). We find the optimal mental account solution and implied risk-aversion parameter ($\gamma = 3.795$).[2] The optimal mental account weights are $w = [0.54, 0.27, 0.19]$. The expected return of the optimal mental account is 10.23%.

Now consider an investor who sets her threshold return higher, at -5%, but is willing to accept a higher probability, 0.15, of failing to reach it. The implied level of risk-aversion for this investor is 2.7063, and her portfolio has a lower allocation to the low-risk security and higher allocations to the medium and high-risk securities. The expected return of this mental account is 12.18%, higher than the earlier one. Last, consider an investor who aims for upside potential and wants a high expected return. This investor sets her threshold return at a relatively low level, -15%, and is willing to accept a 0.20 probability of failing to reach it. The risk aversion parameter, γ, of this investor is 0.8773. She allocates even more to the medium and high-risk securities, and takes a short position in the low-risk security. The expected return of the mental account is 26.35%, much higher than the other two. These cases are summarized below.

[2] We determine the implied risk-aversion by solving problem (2) and then find the γ in the solution to problem (1) that gives the same optimal portfolio weights. Note that this is feasible on account of the known equivalence between the two problems.

Threshold (H)	Probability (α)	Expected return (%)
−10%	0.05	10.23
−5%	0.15	12.18
−15%	0.20	26.35

Now we add options and structured products to this initial set of securities, and analyze investor choices.

3. Optimization with options and structured products

Portfolio optimization when the joint distribution of the returns of securities is fully described by the mean and variance of returns has been extensively analyzed and widely implemented. In this section, we extend the set of securities by adding options and structured products. That addition creates non-zero higher moments of portfolio returns even when the added options and structured products are mere combinations of the initial set of (multivariate normal) securities. Hence, mean–variance optimization is no longer feasible.

The optimization problem in Eq. (2) requires integration over the entire multivariate distribution of returns of all securities, including options and structured products. The problem is then restated as follows:

$$\max_{w} \int_{u \in U} [w' r(u)] \cdot p[r(u)] \cdot \prod_{i=1}^{n} dr_i(u) \tag{3}$$

s.t.

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \tag{4}$$

$$\int_{u:[w' r(u)]] < H} p[r(u)] \cdot \prod_{i=1}^{n} dr_i(u) \leq \alpha \tag{5}$$

where $r(u) \in R^n$ is a vector of returns in state u of the state space U. The constraints above require that an investor is fully invested and the risk criterion is satisfied. In addition, we disallow short-selling. Here, $w \in R^n$ is the vector of mental account weights for the n securities in the mental account, and $p[r(u)]$ is the multivariate probability of occurrence of the vector of returns $r(u)$.[3]

Maximization of the function above is numerically intensive, and the hyper-surface described by the objective function in n-space has many local optima because of the constraints, so finding the global optimum may prove elusive. Hence we use a combination of searching the state-space ($\mathbf{r} \in U$), weight-space (w), and numerical hill-climbing. The computational algorithm to solve this problem is described in Appendix.

Next, we use the computational algorithm to examine the place of options and structured products in portfolios.

3.1. Put options

Investors want downside protection in mental accounts devoted to their retirement goal. Puts have a place in such mental accounts. We expand the initial set of three securities by adding a fourth, a put on the medium-risk security. The medium-risk security is analogous to a stock index, and its put is analogous to an index put. We also impose short-sale constraints. We assume a one-year horizon for this mental account, so investors buy puts with one year to expiration. We set the strike price K within a range of 0.8–1.2 times the value of the underlying security, thereby covering options from out-of-the-money to in-the-money. We assume that the risk free rate is 3%, a rate lower than the 5%, expected return of the low-risk security. This rate is exogenously supplied with no loss of generality. The option volatility is the standard deviation of return of the medium-risk security.

We use the risk-neutral probability measure in pricing options. We price the options on the medium-risk security, using the Black–Scholes model, for an underlying price of 1 to get its current price, denoted as P_0. Then, using the returns in the state-space of the medium-risk security, we compute the payoff of the option, denoted as $P(u) = \max[0, K-(1+r_2(u))], \forall u \in U'$, where r_2 is the return on the medium-risk security (Security 2 in our analysis). The return on the put (Security 4) in each state u is $r_4(u) = [P(u) - P_0]/P_0$. The upside return is as high as $(K-P_0)/P_0$, but the downside is a total loss when the option ends up out-of-the-money. We augment the state-space U' of returns to the base assets (Securities 1–3) with the vector r_4 to get the full state-space U (Securities 1–4). Our procedure automatically ensures that the put option is also appropriately correlated with the other securities.

Assuming multivariate normality of the returns of the initial set of three securities, and using the data as in Section 2.1, we maximize the expected return of the mental account with puts as the fourth security. For computational speed, we use a sparse support of only 50 grid points for each security weight to generate a discrete multivariate distribution grid, resulting in a small change in security weights from the solution based on continuous multivariate normality, with little

[3] A similar problem set up is used in Janssen (2008) but the only choice securities are Arrow–Debreu securities.

attenuation in maximized expected return (less than 5 bps). The results are in Table 1. Panel *A* in the table shows the basic return statistics of puts at various strike prices. The expected returns of puts range from −27% to −39%. The variance of returns is very large. For example, a one-standard deviation of return for the at-the-money option is 128%($=\sqrt{163\%}$, square-root of the variance reported in Table 1). Put options are unlikely to be attractive by themselves because the expected returns of puts are negative and the variance of their returns is large. Puts might be useful however, in conjunction with the other securities in mental accounts.

We optimize the downside protection mental account making sure that we satisfy the constraint that the probability of failing to reach a return $H=-10\%$ is lower than $\alpha=0.05$. The portfolio weights we obtain are in Panel *B* of Table 1. The return moments of the optimized mental account are also presented. We see that puts are not in the optimal mental account. Increasing the acceptable probability of failing to reach $H=-10\%$ to $\alpha=0.10$ does not change the result, as it only diminishes the need for puts.

Puts are mainly useful when the threshold return *H* is set relatively high. Consider $H=0\%$ and $\alpha=0.05$ in Panel D of Table 1. There is no feasible solution when only the initial three securities are available. However, in-the-money puts at a strike price of 1.2 allow a feasible solution in this mental account, containing an allocation of 0.15 to puts.

Finally, consider investors who raise the threshold returns *H* beyond zero to $H=1\%$ or $H=2\%$ while α remains at 0.05. We can see from Panels *E* and *F* of Table 1 that as *H* is raised, investors choose puts that are further in-the-money.

Table 1
Optimal mental account allocations when put options are included as the fourth security. The top panel of the table shows basic statistics of the returns of puts. The threshold return is varied in the range $H=\{-10\%,-5\%,0,+1\%,+2\%\}$ and the maximum probability of failing to reach these thresholds is $\alpha=0.05$. The risk free rate is set to 3%. The expected return of the mental account is maximized subject to the constraint that the maximum probability of failing to reach the threshold *H* is α. We report the weights in each of the four securities: $\{w_1,w_2,w_3,w_4\}$ (corresponding to the low, medium, and high-risk securities, and the put), and the expected return and standard deviation of the optimized mental account. The first line of the table shows the mental account when puts are not available and the remaining lines show it when they are.

Strike	Price	Mean	Variance
Panel *A*: Put Return Statistics			
0.8	0.0086	−0.3114	11.8849
0.9	0.0277	−0.3941	3.5614
1	0.0646	−0.3847	1.6320
1.1	0.1204	−0.3358	0.9379
1.2	0.1922	−0.2730	0.5980
1.3	0.2752	−0.2122	0.3962
1.4	0.3649	−0.1617	0.2665

	LowRisk	MedRisk	HighRisk	LongPut		Portfolio return moments			
Strike	w_1	w_2	w_3	w_4	$Pr[r<H]$	Mean	Std. Dev.	Skewness	Kurtosis
Panel *B*: $H=-10\%,\alpha=0.05$									
No put	0.5871	0.2052	0.2077	–	0.0498	0.1018	0.1226	0.0000	0.0000
0.8	0.5773	0.2058	0.2139	0.0029	0.0497	0.1020	0.1235	0.0335	−0.0184
0.9	0.5862	0.2054	0.2082	0.0003	0.0496	0.1018	0.1227	0.0031	−0.0020
1	0.5873	0.2034	0.2078	0.0000	0.0496	0.1017	0.1225	0.0001	0.0000
1.1	0.5796	0.2112	0.2081	0.0011	0.0498	0.1018	0.1229	0.0046	0.0001
1.2	0.5897	0.2011	0.2091	0.0000	0.0499	0.1019	0.1229	0.0000	0.0000
Panel *C*: $H=-5\%,\alpha=0.05$									
No put	0.6964	0.2008	0.1028	0.0000	0.0498	0.0806	0.0793	0.0000	0.0000
0.8	0.6986	0.1998	0.1016	0.0000	0.0491	0.0803	0.0788	0.0008	−0.0010
0.9	0.6982	0.2002	0.1016	0.0000	0.0491	0.0803	0.0788	0.0003	−0.0002
1	0.6971	0.2000	0.1028	0.0001	0.0495	0.0805	0.0792	0.0011	−0.0005
1.1	0.6962	0.2000	0.1035	0.0003	0.0496	0.0806	0.0793	0.0032	0.0000
1.2	0.6977	0.1994	0.1029	0.0000	0.0495	0.0805	0.0792	0.0000	0.0000
Panel *D*: $H=0\%,\alpha=0.05$									
No put	Infeasible: no solution								
0.8–1.1	Infeasible: no solution								
1.2	0.0488	0.7996	0.0043	0.1473	0.0495	0.0433	0.0688	2.4538	6.5319
1.3	0.2459	0.6007	0.0104	0.1430	0.0495	0.0446	0.0422	2.4319	8.7183
Panel *E*: $H=1\%,\alpha=0.05$									
No put	Infeasible: no solution								
0.8–1.2	Infeasible: no solution								
1.3	0.0489	0.7514	0.0043	0.1954	0.0495	0.0372	0.0431	3.6502	16.0972
Panel *F*: $H=2\%,\alpha=0.05$									
No put	Infeasible: no solution								
0.8–1.3	Infeasible: no solution								
1.4	0.0497	0.7015	0.0012	0.2476	0.0496	0.0329	0.0228	6.0207	46.3634

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The expected returns on the optimized mental account decline sharply, but puts remain essential since there are no feasible solutions without them.

When $H=0\%$, investors include a strike 1.2 put in the portfolio with a 0.15 weight. This results in an expected return of 4.33%. The standard deviation of returns (6.88%) is lower than the case when threshold return H is lower. The inclusion of puts leads to mild positive skewness (2.45) in the portfolio, yet excess kurtosis increases quite substantially to 6.53. Interestingly, when a strike 1.3 put is included, the expected return increases to 4.46%, and the standard deviation falls to 4.22%, but the kurtosis rises further to 8.72.

Strike 1.2 puts are no longer sufficient for a feasible solution once we increase the threshold return H to 1%. Strike 1.3 puts are required. The expected return of the optimized portfolio falls to 3.72%, though the 4.31% standard deviation of returns is low. Not surprisingly, the 3.65 skewness becomes more positive, but the 16.10 excess kurtosis becomes much higher. We reach a similar conclusion when $H=2\%$. Now strike 1.4 puts (even more deep in-the-money) are needed. Mean and standard deviation of returns of the optimized portfolio drop to 3.29% and 2.28% respectively. The 6.02 skewness and 46.37 excess kurtosis are markedly higher.

3.2. Call options

Whereas put options are useful in downside protection mental accounts, such as ones dedicated to retirement income, call options are useful in upside potential mental accounts, such as those dedicated to bequests. As with puts, the strike price K is within a range of 0.8–1.2 times the value of the underlying medium-risk security, thereby covering options from in-the-money to out-of-the-money. The risk free rate is 3% as before. The option volatility is the standard deviation of return of the medium-risk security.

We price the call option on the medium-risk security using the Black–Scholes model for the price of 1.0 of the underlying medium-risk security. We denote the current price of the call as C_0. Then using the returns in the state-space of the medium-risk security, we computed the payoff states of the option, denoted $C(u)=\max[0,(1+r_2(u))-K],\forall u \in U'$. The return on the option in each state u is $r_4(u)=[C(u)-C_0]/C_0$. The upside return potential is infinite, but the downside return is a total loss. We augment the state-space U' with the vector r_4 to get the full state-space U. Our procedure automatically ensures that the call option is appropriately correlated with the other securities.

Using the same input data as with the puts in the preceding subsection, we maximize the expected return of the mental account with the call as the fourth security. Panel A of Table 2 shows the basic return statistics of the call at various strike prices. The expected return of the call ranges from 32% to 51%. The variance of return is very large. For example, the one-standard deviation of return for the at-the-money option is $158\%(=\sqrt{249\%}$, square-root of the variance).

Panel B of Table 2 shows that the high expected returns of calls makes them attractive for upside potential mental accounts. We see that the mental account contains a 0.10 or 0.15 allocation to calls that are in-the-money. The allocation is 0.05 when calls are at-the-money, or just out-of-the-money. When calls are out-of-the-money, the portfolio does not include calls. But such calls are needed in the portfolio when the threshold return is set very high. For example, if an investor wants to reach a threshold return of $H=50\%$ with at least a probability of 30% (i.e., $\alpha=0.7$), the optimal portfolio will be invested entirely in the strike 1.1 call (10% out-of-the-money). The expected return of the portfolio is 51.11% and the standard deviation is 220%, with a skewness of 1.64 and excess kurtosis of 2.42.

Comparing a mental account where only the initial three securities are available to one where call options with strike price of 0.8 are added we find that the 0.10 allocation to calls increases the expected return of the mental account by 63 basis points, from 10.18% to 10.81%. There is a corresponding increase in the standard deviation of the portfolio from 12.26% to 12.96%. Using out-of-the-money calls results in a positive skew of the optimized portfolio and also reduces kurtosis.

In Panel C of Table 2 we depict the expected returns of mental accounts with and without call options (at strike $K=0.9$) for different levels of probability of failing to reach the threshold returns $H=-10\%$. We vary the probability α from 0.05 to 0.30 in intervals of 0.025. We see that the mental account efficient frontier is higher with calls than without them for all levels of the risk constraint, α. When α is low, i.e., the risk constraint is stringent, the addition of calls results in gains of about 50 basis points. But when the constraint loosens up, as α increases, gains of over 500 basis points are possible. Thus, investors with low risk aversion stand to gain more from using calls than investors with high risk aversion.

3.3. Combining calls and put options

A safety collar combines a short position in a call and a long position in a put. The collar places a floor under the value of the underlying stock and the premium received for the call offsets some of the premium paid for the put. Table 3 shows the efficient frontier of a mental account where collars are added to the initial three securities. This collar benefits investors who aim for downside protection in this mental account, with a low probability α of not reaching a threshold return H. The desirability of a collar declines and eventually disappears for investors who want a higher expected return and are willing to accept a higher probability of not reaching the threshold return.

An aggressive collar, combining a short position in a put and a long position in a call enables investors to take leveraged positions in securities. Aggressive collars compound gains when securities appreciate, but magnify losses when securities depreciate. Some examples of adding such collars to the initial three securities are shown in Table 4, Panel A. We see that

Table 2
Optimal mental account allocations when call options are added as the fourth security. The top panel of the table shows basic statistics of the returns on the call option. The threshold return is $H=-10\%$ and the permissible probability of failing to reach this threshold is $\alpha=0.05$. The risk free rate is set to 3% and the option maturity is 1 year. The mental account expected return is maximized subject to the constraint that the maximum probability of failing to reach the threshold H is α. We report the weights of each of the four securities: $\{w_1,w_2,w_3,w_4\}$ (corresponding to the low, medium, and high-risk securities, and the call), and the expected return and standard deviation of the optimized mental account (Panel B). The first line of the table shows the mental account when calls are not available and the remaining lines when they are.

Strike	Call price	Call mean	Call variance
Panel A: Call Return Statistics			
0.8	0.2322	0.3173	0.6582
0.9	0.1543	0.4050	1.2599
1	0.0941	0.4844	2.4928
1.1	0.0529	0.5111	4.8543
1.2	0.0277	0.4362	8.8926

	LowRisk	MedRisk	HighRisk	LongCall		Portfolio return moments			
Strike	w_1	w_2	w_3	w_4	$Pr[r<H]$	Mean	Std. Dev.	Skewness	Kurtosis
Panel B: $H=-10\%,\alpha=0.05$									
No call	0.5871	0.2052	0.2077	–	0.0498	0.1018	0.1226	0.0000	0.0000
0.8	0.7419	0.0027	0.1524	0.1030	0.0496	0.1081	0.1296	0.1186	−0.1196
0.9	0.7968	0.0008	0.1005	0.1019	0.0496	0.1063	0.1395	0.3934	−0.1643
1	0.6978	0.1000	0.1498	0.0514	0.0496	0.1072	0.1403	0.4424	0.0592
1.1	0.7954	0.0032	0.1483	0.0531	0.0497	0.1043	0.1546	0.9297	1.0514
1.2	0.5948	0.1995	0.2017	0.0040	0.0495	0.1019	0.1244	0.0817	0.0479

$\alpha=Prob[r<H]$	Baseline portfolio	Baseline plus calls	Gain in exp. return
Panel C: $H=-10\%$, strike $K=0.9$			
0.050	0.1018	0.1063	0.0045
0.075	0.1141	0.1183	0.0042
0.100	0.1266	0.1313	0.0047
0.125	0.1414	0.1506	0.0092
0.150	0.1582	0.1709	0.0126
0.175	0.1764	0.1964	0.0200
0.200	0.1981	0.2420	0.0439
0.225	0.2264	0.2918	0.0655
0.250	0.2500	0.3043	0.0543
0.275	0.2500	0.3129	0.0629
0.300	0.2500	0.3284	0.0784

Table 3
Mental account efficient frontiers with collars and without them. Risk is represented by the probability α of failing to meet the return threshold $H=0\%$ in the leftmost column of the table. The next two columns show the expected return of the mental account. We vary the probability α of failing to reach the threshold returns from 0.05 to 0.30 in increments of 0.025. The collar combines a short position in a call at strike price 1.60 with a long position in a put at strike price 1.20.

$\alpha=Prob[r<H]$	Baseline exp return	Return with collars
0.050	Infeasible	0.043
0.075	Infeasible	0.044
0.100	Infeasible	0.051
0.125	0.069	0.069
0.150	0.079	0.079
0.175	0.088	0.088
0.200	0.100	0.100
0.225	0.117	0.117
0.250	0.142	0.142
0.275	0.186	0.186
0.300	0.215	0.215

adding a short collar with a weight of 0.05 in the mental account results in an increase in expected return of 40–50 basis points for a given combination of α and H. Allowing for a less risk-averse investor, as in Panel B, results in an additional pick up of 50–100 basis points.

We summarize the results from Tables 1–4. (a) Puts become essential in mental accounts as threshold H increases and probability α decreases. (b) Calls become increasingly attractive when H decreases and α increases. (c) Because calls have a positive expected return under the physical probability measure, shorting calls is not desirable unless they are being used

Table 4
Optimal mental account allocations with "aggressive" collars added as the fourth security. These are short collars—a mental account combining a long call option at strike K_c and a short put option at strike price K_p, where $K_c \geq K_p$. The threshold return is H and the permissible probability of failing to reach this threshold is α. The risk free rate is set to 3%, and the collar maturity is 1 year. We report the weights in each of the four securities: $\{w_1,w_2,w_3,w_4\}$ (corresponding to low, medium, and high-risk securities, and the collar), and the expected return, standard deviation, skewness and kurtosis of returns of the optimized mental account. The first line of the table shows the mental account when collars are not allowed and the remaining lines when they are.

	LowRisk	MedRisk	HighRisk	Collar	Portfolio return moments			
Strike	w_1	w_2	w_3	w_4	Mean	Std. Dev.	Skewness	Kurtosis
Panel A: $H=-10\%,\alpha=0.05$								
No collar	0.5871	0.2052	0.2077	–	0.1018	0.1226	0.0000	0.0000
$K_c=1.0,K_p=1.0$	0.5986	0.1998	0.2001	0.0015	0.1035	0.1241	0.0000	0.0000
$K_c=1.05,K_p=0.95$	0.5991	0.1983	0.2002	0.0015	0.1027	0.1234	0.0069	0.0078
$K_c=1.0,K_p=0.8$	0.7984	0.0014	0.1494	0.0507	0.1060	0.1363	0.3236	0.1361
$K_c=0.9,K_p=0.7$	0.6880	0.1007	0.1551	0.0562	0.1062	0.1302	0.1630	−0.0727
$K_c=0.8,K_p=0.7$	0.7354	0.0068	0.2039	0.0539	0.1056	0.1225	0.0215	−0.0148
Panel B: $H=-15\%,\alpha=0.10$								
No collar	0.1466	0.4515	0.4019	–	0.1530	0.2363	0.0000	0.0000
$K_c=1.0,K_p=1.0$	0.1949	0.4044	0.3979	0.0027	0.1562	0.2398	0.0000	0.0000
$K_c=1.05,K_p=0.95$	0.1493	0.4497	0.4003	0.0006	0.1537	0.2375	0.0016	0.0018
$K_c=1.0,K_p=0.8$	0.5992	0.0004	0.2998	0.1006	0.1617	0.2611	0.3592	0.1565
$K_c=0.9,K_p=0.7$	0.4412	0.1008	0.3530	0.1050	0.1633	0.2476	0.1355	−0.0537
$K_c=0.8,K_p=0.7$	0.4440	0.0499	0.3530	0.1531	0.1643	0.2452	0.0545	−0.0491

to subsidize the cost of protective puts. Hence, safety collars, i.e., short calls, long puts are favored when H increases, and α decreases. Conversely, puts are not needed when α is high (as in Table 3), and because shorting calls is expensive, no calls are used either, and the benchmark portfolio without collars suffices. (d) Aggressive collars (Table 4) are only useful when the threshold H is low, and α is high.

3.4. Alternate risk criteria: expected shortfall

So far we have measured risk by the probability α that the return of a mental account will fail to reach a threshold return H. This risk measure has been faulted for overlooking the shape of the tails of a portfolio's return distribution. Missing a return threshold when the distribution has a fat tail has more serious consequences than when the tail is not as fat. Moreover, risk as the probability of failing to reach a threshold level is not included in the class of coherent risk measures, as defined in Artzner et al. (1999).

Yet our optimization algorithm can be extended to other risk measures such as "expected shortfall" (ES for short). ES is a coherent risk measure and explicitly accounts for the shape of the tail of the distribution beyond the threshold. ES is also known as CVaR (conditional VaR) and a detailed analysis of various portfolio problems with CVaR constraints was developed in Krokhmal et al. (2002) though derivatives were not considered. Investors employing ES as their risk measure maximize the expected return of portfolios, subject to the constraint that the expected return $E(r)$ below the threshold H is bounded below by a pre-specified limit L, i.e., $E[r|r<H] \geq L$, or

$$\frac{\int_{-\infty}^{H} r \cdot f(r)\, dr}{\int_{-\infty}^{H} f(r)\, dr} \geq L \tag{6}$$

We apply this risk measure with the mean and covariance matrix used earlier, and let investors buy put options in addition to the initial three securities. As before, the underlying security for the put is the medium-risk security. The threshold return is $H=-10\%$, and we set the boundary level of ES at $L=-13\%$. The results are presented in Table 5.

Comparing the base case in Table 1 with the ES case (with no puts), we see that the optimal portfolio for this mental account is more conservative when the ES constraint is applied than when the threshold return constraint is applied; the weight of the high-risk security is lower in ES and the weight of the low-risk security is higher, although this depends on the specific threshold of expected shortfall. We can also see that the inclusion of puts in the mental account is optimal for strike prices close to the money (in the 0.9–1.1 range). The expected return of the portfolio is 9.72% when puts with a strike price of 0.9 are used, higher than the 8.33% expected return when puts are not available. Both mental accounts satisfy the expected shortfall constraint. The standard deviation of returns of the mental account with puts is higher than the one without puts, but the higher standard deviation is offset by positive skewness and negative excess kurtosis. Also, the availability of puts leads to lower allocations to the low-risk security and a higher allocation to the medium-risk security. Puts do not add value to the mental account when they are available only in strike prices that are deep in-the-money or deep out-of-the-money. Overall, the availability of puts allows investors to construct mental accounts with lower risk,

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Table 5
Optimal mental account allocations with expected shortfall as the risk criterion. Puts are added as the fourth security. The top panel of the table shows basic statistics of the returns on the put option. The threshold return is $H=-10\%$ and the permissible level of expected shortfall is $L=-13\%$. The risk free rate is set to 3%, and the option maturity is 1 year. The mental account expected return is maximized subject to the constraint that the expected shortfall should not be greater than L. We report the weights in each of the four securities: $\{w_1,w_2,w_3,w_4\}$ (corresponding to Low, Medium, and high-risk securities, and the put), and the expected return, standard deviation, skewness and kurtosis of returns of the optimized mental account. The first line of the table shows the mental account when puts are not available and the threshold constraint is applied. The second line shows the mental account when puts are not available and the expected shortfall constraint is applied. The remaining lines show the mental account when they are. The initial case is the same as in the previous tables, i.e., the ES constraint is not used. The "No put" case is the one where puts are not available. In all others cases the strike price of the put is provided.

	LowRisk	MedRisk	HighRisk	LongPut	Portfolio return moments			
Strike	w_1	w_2	w_3	w_4	Mean	Std. Dev.	Skewness	Kurtosis
$H=-10\%$, $L=-13\%$								
Base case	0.5871	0.2052	0.2077	–	0.1018	0.1226	0.0000	0.0000
No put	0.6468	0.2487	0.1045	–	0.0833	0.0854	0.0000	0.0000
0.8	0.6468	0.2487	0.1045	0.0000	0.0833	0.0854	0.0000	0.0000
0.9	0.1462	0.7241	0.1066	0.0231	0.0972	0.1435	0.4918	−0.1564
1.0	0.0042	0.8491	0.1022	0.0446	0.0935	0.1510	0.6928	0.1146
1.1	0.1976	0.6515	0.1028	0.0481	0.0846	0.1155	0.6361	0.3977
1.1	0.6469	0.2499	0.1029	0.0003	0.0830	0.0850	0.0002	0.0001

measured by expected shortfall, higher expected returns, or a combination of the two. These results tell us how to optimize mental accounts with options, the impact of options on the weights of securities in mental accounts, and the moments of the distributions of returns of mental accounts.

Many derivatives are available beyond plain calls and puts. We examine some in the next section.

4. Structured products

4.1. Capital guaranteed notes

Capital guaranteed notes combine a return floor and a chance for higher returns. Like puts and collars, they are suited for investors' downside protection mental accounts.[4] Capital guaranteed notes have three main features which can be varied to suit investors' preferences: a return floor, usually zero but sometimes higher, a participation rate in the index, such as 70% of the appreciation of the S&P 500 Index, and a cap on returns, such as at 20%.[5]

A capital guaranteed note of notional value \$1, floor F, participation rate y, and cap C may be priced as

$$CGN = e^{-rT}(1+F)+y[\text{Call}(1+F)-\text{Call}(1+C)] \tag{7}$$

where Call(K) is the Black–Scholes premium of a call option with a strike of K and a \$1 initial value of the underlying index. The risk free rate is r, and the maturity of the note is T years. The equation above for the fair price of a capital guaranteed note shows the fair price for the capped upside payoff (second term in the equation above) at which the note will trade above the present value of the notional (the first term). The return on the note is computed by dividing the payoff of the note by the price of the capital guaranteed note.

Holding the threshold return as before at $H=-10\%$, and the probability of failing to reach it at no more than $\alpha=0.05$, we examine the potential roles of various capital guaranteed notes in mental accounts. The results are shown in Table 6.

The first capital guaranteed note had a 0% floor, a 50% participation rate, and no cap. Its fair price is \$1.0175. The weight of this note in the optimized mental account is 0.66. The expected return of the mental account is 10.78%. The expected return of the mental account is higher than the one without this note by 60 bps as seen in a comparison of the first two lines in Table 6.

The second capital guaranteed note has a 3% floor, a 50% participation rate, a 20% cap, and a price of \$1.0257. (Note that the effective floor is 2.9% since the note is priced at \$1.0257 rather than at \$1.) Investors in this note sacrifice some upside potential for greater downside protection. The weight of this note in the mental account is 0.59. The expected return of the mental account is 10.69%. This note is inferior to the one with the zero-floor and no cap for an investor with a threshold of −10% and a 0.05 probability of missing it. Therefore, even though investors like the higher floor, they are 9 bps worse-off in expected return compared to the first note. The third note is created by modifying the second note, dropping the floor to 2.5% and raising the cap to 25%. Its price is \$1.0260. This note has a weight of 0.60 in the optimized mental account, which

[4] There are many variations of these products, under many names, such as equity participation notes, equity indexed annuities, and stock index insured accounts. See http://riskinstitute.ch/00011311.htm.

[5] Milevsky and Kyrychenko (2008) described portfolio allocations when capital guaranteed notes are included in the set of available securities; Boyle and Tian (2008) analyzed these securities and specified the best form that they should take.

Table 6
Optimal mental accounts with capital guaranteed notes. The expected return of the mental account is maximized subject to the constraint that the maximum probability of failing to reach the threshold $H = -10\%$ is $\alpha = 0.05$. We report the weights in each of the four securities: $\{w_1, w_2, w_3, w_4\}$ (corresponding to low, medium, and high-risk securities, and the capital guaranteed note indexed to the medium-risk security), and the expected return and standard deviation of the optimized mental account. The first line of the table shows the mental account when capital guaranteed notes are not allowed. The next three lines report results when capital guaranteed notes are allowed and are fairly priced by the seller. The next two lines show the results when sellers of capital guaranteed notes add a premium to the price. The last line presents a mental account that includes a risk free note with a 4.5% return. (We have assumed a 3% risk-free rate elsewhere in the table and note that the risk-free security is not included in the mental account when its rate is 3%.)

	LowRisk	MedRisk	HighRisk	Annuity		Portfolio return moments			
Strike	w_1	w_2	w_3	w_4	$Pr[r<H]$	Mean	Std. Dev.	Skewness	Kurtosis
$H = -10\%$, $\alpha = 0.05$									
No CGN	0.5871	0.2052	0.2077	–	0.0498	0.1018	0.1226	0.0000	0.0000
Floor = 0% Upside = 50% Cap = None	0.0415	0.1011	0.2024	0.6550	0.0498	0.1078	0.1318	0.1918	−0.0013
Floor = 3% Upside = 50% Cap = 20%	0.0010	0.2069	0.2053	0.5867	0.0495	0.1069	0.1272	0.0187	−0.0957
Floor = 2.5% Upside = 50% Cap = 25%	0.0000	0.2008	0.2022	0.5970	0.0498	0.1081	0.1293	0.0470	−0.1386
Floor = 2.5% Upside = 50% Cap = 25%, Prem = 8%	0.0977	0.2013	0.2007	0.5003	0.0496	0.1044	0.1256	0.0359	−0.1024
Floor = 2.5% Upside = 50% Cap = 25%, Prem = 17.8%	0.1942	0.2003	0.2016	0.4039	0.0495	0.1018	0.1233	0.0253	−0.0695
Floor = 4.5% Upside = 100% Cap = 4.5%	0.3430	0.2494	0.2027	0.2049	0.0496	0.1020	0.1229	0.0000	0.0000

has an expected return is 10.81%, a gain of 21 bps over the second note, and better than the first note by 3 bps. All these notes roughly improve the portfolio by 50–60 bps over the case with no capital guaranteed notes.

Recall the earlier conclusion of Hens and Rieger (2008) that capital guaranteed notes have no place in optimal mean–variance and expected utility portfolios but are valuable additions in a prospect theory setting or an expected utility framework with aspirational motives. Complementing this, we also find that fairly priced capital guaranteed notes have a place in optimized threshold-based portfolios.

Further, recall the earlier finding of Henderson and Pearson that capital guaranteed notes are overpriced by eight percent. Extreme overpricing would surely eliminate all the benefits of capital guaranteed notes for investors. But by how much can capital guaranteed notes be overpriced before their benefits are dissipated? The fifth line in the table shows that capital guaranteed notes are included in optimal mental accounts even when they are overpriced by 8%. Indeed, the benefits of capital guaranteed notes are not dissipated until overpricing reaches 17.8%.

4.2. Barrier-M note

Barrier-M notes are also suited for downside protection mental accounts since their minimum return is zero. The notes pay a return equal to the absolute value of the underlying security's return subject to a barrier level, say $M = 25\%$. The payoff diagram of this security looks like an "M", hence the name. They pay zero if the absolute value of the return exceeds the barrier. The returns of the notes are capped at the barrier, making them less suitable for upside-potential mental accounts.

We assume that the underlying security for the note, as before, is the medium-risk security. The barrier-M note is now the fourth security, joining the initial three. The return r_4 on the barrier-M note's notional value is represented as

$$r_4 = \begin{cases} |r_2| & \text{if } |r_2| \le 0.25 \\ 0 & \text{if } |r_2| > 0.25 \end{cases} \tag{8}$$

We stay away from predicting returns. We assume that investors assess these correctly, i.e., there are no bargain securities.

Barrier-M notes will sell at a premium to their notional because of the various options that make up the payoff profile of the note. In total, a zero-coupon bond plus six options are needed to replicate a barrier-M note. Assuming an underlying index at value $1, the options needed are as follows:

1. A long call at strike $1.
2. A long put at strike $1.

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Table 7
Optimal portfolio allocations when Barrier-M notes are added to the initial three securities. The expected return of a mental account is maximized subject to the constraint that the maximum probability of failing to reach the threshold $H=-10\%$ is $\alpha=0.05$. We report the weights in each of the four securities: $\{w_1,w_2,w_3,w_4\}$ (corresponding to Low, Medium, and high-risk securities, and the barrier note), and the expected return and standard deviation of the optimized mental account. The first line of the table shows the mental account when Barrier-M notes are not allowed and the second line when they are. The expected return of the note is 7.51% and its standard deviation is 7.06%.

	LowRisk	MedRisk	HighRisk	Long note		Portfolio return moments			
Strike	w_1	w_2	w_3	w_4	$Pr[r<H]$	Mean	Std. Dev.	Skewness	Kurtosis
$H=-10\%$, $\alpha=0.05$									
No note	0.5871	0.2052	0.2077	–	0.0498	0.1018	0.1226	0.0000	0.0000
Barrier: ± 0.25	0.0009	0.0426	0.2635	0.6929	0.0495	0.1237	0.1405	0.0131	0.0048

3. A short call at strike $(1+M)$.
4. A short put at strike $(1-M)$.
5. M short cash-or-nothing calls at strike $(1+M)$.
6. M short cash-or-nothing puts at strike $(1-M)$.

Cash-or-nothing (CON) options pay off $1 if the option is in-the-money, else they pay off zero. These six options are easily evaluated. The first four options are priced using the Black–Scholes formula. The last two options are digital options and have known pricing equations, which are

$$CON_{call}[\text{Strike}=1+M]=e^{-rT}N\left[\frac{\ln(1/(1+M))+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right] \tag{9}$$

$$CON_{put}[\text{Strike}=1-M]=e^{-rT}N\left[-\left(\frac{\ln(1/(1-M))+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right)\right] \tag{10}$$

The risk free rate is denoted as r, the maturity is T and σ is the volatility of returns on the underlying security. The net cost of these options plus the notional value of the note will comprise its price on which the return will be computed. For the note analyzed here, with $M=0.25$, the net cost of the options is 0.0838, or 8.38% of the notional. Adding this cost to the present value of the zero-coupon bond results in a price for the M-note of 1.0542. The returns on the M-note are the ratio of the payoff to the price of the note.

The effect on risk and expected returns from adding barrier-M notes into mental accounts is in Table 7. The note has appealing mean–variance properties. At a barrier of $\pm 25\%$ the expected return of the note is 7.72% and the standard deviation of return is 7.25%. Hence, it has a slightly higher risk and expected return than the low-risk security, though its return distribution is not normal. The optimized mental account contains a 0.69 allocation to the Barrier-M note. The expected return of the mental account increases substantially, from 10.18% to 12.37%. The mental account is now positively skewed, but there is also a mild increase in kurtosis. Since the Barrier-M note has a minimum return of zero, it takes the place of the low-risk security to which no allocation is made. There is a substantial reduction in the allocation to the medium-risk security, and a small increase in the allocation to the high-risk security. Overall, the risk-return profile of mental accounts is significantly improved with the addition of Barrier-M notes.

We can extend the analysis by considering multivariate distributions that are non-normal and constraints on short-selling.

5. Non-normal optimization with copulas

Risk-return tradeoffs in the mean–variance framework approximate the risk-return tradeoff in many types of utility functions (see Levy and Markowitz, 1979). The risk-return tradeoff in Eq. (2) is more general than the tradeoff in the mean–variance framework and is applicable to multivariate return distributions of any kind, beyond returns from normal distributions or investors with quadratic utility.

We present a method for optimizing mental accounts that is applicable to general multivariate return distributions. Our implementation relies on modeling the joint distribution of returns using copulas (see Sklar, 1959, 1973; Frees and Valdez, 1998). Copula techniques enable the representation of the joint distribution as a composite of a correlation function and separate marginal distributions for each security. For example, the initial three securities we have considered earlier might follow a multivariate Student-t distribution rather than a multivariate normal distribution. A copula is a function that "couples" together marginal distributions into a joint distribution. Denote the returns on n securities as $r_i, i=1\ldots n$, each with distribution function $F_i(r_i)$. The joint distribution is denoted $F(\mathbf{r})\equiv F(r_1,r_2,\ldots,r_n)$ and may be represented by means of a copula function $C[F_1(r_1),F_2(r_2),\dots,F_n(r_n)]$ as follows:

$$F(r_1,r_2,\dots,r_n) = C[F_1(r_1),F_2(r_2),\dots,F_n(r_n)] = C(u_1,u_2,\dots,u_n) \tag{11}$$

where $u_i = F_i(r_i)$. Sklar (1959) showed that a copula must exist for any joint distribution. To see the decomposition of the joint distribution into correlations and marginal distributions, differentiate the equation above for the joint distribution to get the joint probability density function, i.e.,

$$\begin{aligned} f(r_1,\dots,r_n) &= \frac{\partial^n F}{\partial r_1 \dots \partial r_n} = \frac{\partial C}{\partial u_1 \dots \partial u_n} \times \frac{\partial u_1}{\partial r_1} \times \dots \times \frac{\partial u_n}{\partial r_n} \\ &= c(u_1,\dots,u_n) \times f_1(r_1) \times \dots \times f_n(r_n) = c(u_1,\dots,u_n) \prod_{i=1}^{n} f_i(r_i) \end{aligned}$$

Therefore, the joint density function $f(r_1,\dots,f_n)$ is decomposed into a copula density $c(u_1,\dots,u_n)$ and a product of marginal densities: $f_1(r_1)\dots f_n(r_n)$.

Many different copulas may be implemented for the same joint distribution; for examples, see Frees and Valdez (1998). In this paper, we use the Gaussian copula, one of the simplest and most widely used in practice. The Gaussian copula is defined as

$$C_\rho(u_1,\dots,u_n) = \Phi[\Phi^{-1}(u_1),\dots,\Phi^{-1}(u_n)]$$

where $\Phi(\cdot)$ is the normal distribution function, and $\rho \in R^{n\times n}$ is the correlation matrix of returns. The Gaussian copula density is given as follows (see Sklar, 1959):

$$c_\rho(u_1,\dots,u_n) = \frac{\partial^n C_\rho[F_1(r_1),F_2(r_2),\dots,F_n(r_n)]}{\partial r_1 \dots \partial r_n} = \frac{f(r_1,\dots,r_n)}{\prod_{i=1}^n f_i(r_i)} = \frac{1}{\sqrt{|\rho|}} \exp\left[-\frac{1}{2}\mathbf{r}'(\rho^{-1} - \mathbf{I})\mathbf{r}\right]$$

where $\mathbf{r} = [r_1,\dots,r_n]'$ and $\mathbf{I}$ is the identity matrix. $|\rho|$ is the determinant of the correlation matrix. Putting it all together we may write the joint density function of security returns as

$$f(\mathbf{r}) = \frac{1}{\sqrt{|\rho|}} \exp\left[-\frac{1}{2}\mathbf{r}'(\rho^{-1} - \mathbf{I})\mathbf{r}\right] \times \prod_{i=1}^{n} f_i(r_i) \tag{12}$$

We then use this density function to optimize mental accounts where the multivariate return distribution composed of marginal densities $f_i(r_i)$ that may come from varied distributions. The same function may be used to determine the probability of failing to reach thresholds when investors choose mental account allocations $w = [w_1,\dots,w_n]'$. In the subsequent examples, we will assume that the marginal densities $f_i(r_i)$, $i = 1\dots n$ are chosen as Student-t. We vary the tail-fatness of the distribution by varying its degrees of freedom, thereby varying the probability of failing to reach thresholds. Recall that as the degrees of freedom declines, the t-distribution becomes increasingly fat-tailed. When the degrees of freedom are very large, the distribution approaches normality.

The extended optimization problem of Eq. (2) with short-selling constraints is then stated as

$$\max_{w \in R^n} \quad E[w'\mathbf{r}] \equiv \int_{\mathbf{r}} [w'\mathbf{r}] \cdot f(\mathbf{r}) \cdot d\mathbf{r}, \tag{13}$$

$$\text{s.t.} \quad w'\mathbf{1} = 1 \tag{14}$$

$$w \ge L_l \tag{15}$$

$$w \le L_u \tag{16}$$

$$\text{Prob}[w'\mathbf{r} \le H] \le \alpha \tag{17}$$

The upper and lower bound vectors for mental account weights are $L_u, L_l \in R^n$. Here $L_l \le w \le L_u \in R^n$ is the vector of constrained mental account weights. Given the density function in Eq. (12), we can compute the expected return ($E[w'\mathbf{r}]$) as well as the probability of not reaching thresholds. Using the same copula-generated density we also check if the constraint $\text{Prob}[w'\mathbf{r} \le H] \le \alpha$ is satisfied. We optimize numerically over the space of values of w. Illustrative results are shown in Table 8.

As the degrees of freedom of the Student-t distribution decline, the tails of the joint distribution become fatter and the probability of failing to reach the threshold return increases. The optimal mental account becomes more conservative in order to meet the constraint that the probability of failing to reach the threshold level H not exceed α. An increasing weight of the low-risk security and a decreasing weight of the high-risk security are evident. A corresponding decrease in the expected return of the optimized mental account is also evident, because risk reduction in the face of increasing tail-fatness calls for sacrificing returns. These results complement those of Campbell et al. (2001) where the optimization is performed over the historical empirical distribution of returns.

Table 8
Optimal mental-account allocations when the distribution of joint returns is Student-t. We optimize the mental account when the returns distributions of the initial three securities are multivariate Student-t, with declining degrees of freedom (*dof* in the table) (i.e., increasing tail fatness). When $dof = \infty$, the joint distribution converges to the limiting multivariate normal. We maximize the expected return of the mental account subject to the constraint that the maximum probability of failing to reach the threshold return H is α. The threshold return is $H = -10\%$ and the permissible probability of failing to reach this threshold is $\alpha = 0.05$. We report the weights in each of the three securities: $\{w_1, w_2, w_3\}$ and the expected return of the optimized mental account. The weight of the low-risk security is w_1 that of the medium-risk security is w_2 and of the high-risk security is w_3.

	Risk of security			
	Low	Medium	High	
Dof	w_1	w_2	w_3	$E(r)\%$
1000	0.58	0.21	0.21	10.20
50	0.59	0.21	0.20	10.02
10	0.60	0.25	0.15	9.30
5	0.79	0.06	0.15	8.24

Table 9
Optimal mental-account allocations when at-the-money calls are allowed in the mental account, and the returns are based on a multivariate student-t distribution. The expected return of the mental account is maximized subject to the constraint that the maximum probability of failing to reach the threshold $H = -10\%$ is $\alpha = 0.05$. We report the weights in each of the four securities: $\{w_1, w_2, w_3, w_4\}$ (corresponding to Low, Medium, and high-risk securities, and the call option), and the return moments of the optimized mental account. The first line of the table shows the mental account when call options are not available. Note that as the degrees of freedom (*dof*) of the Student-t distribution decrease, the tails become increasingly fat.

	LowRisk	MedRisk	HighRisk	Long call		Portfolio return moments			
Strike	w_1	w_2	w_3	w_4	$Pr[r < H]$	Mean	Std. Dev.	Skewness	Kurtosis
$H = -10\%$, $\alpha = 0.05$									
$dof = \infty$	0.6978	0.1000	0.1498	0.0514	0.0496	0.1072	0.1403	0.4424	0.0592
$dof = 10$	0.7398	0.1017	0.1036	0.0549	0.0497	0.0990	0.1445	0.7937	1.7333
$dof = 5$	0.9006	0.0315	0.0066	0.0612	0.0498	0.0666	0.1714	2.6490	8.5712

5.1. Using options with copula-based returns

Our framework enables many extensions to mean–variance optimization, such as the simultaneous use of copulas and options. One example is in Table 9 where we combine call options and a Student-t copula.

Options are priced using risk-neutral probabilities, but portfolios are optimized using real-world probabilities. To price a call option under this Student-t copula we change the drift so the expected return on the medium-risk security underlying the call is the risk free rate. Then we price the call using the modified (risk-neutral) probability distribution. Next, we compute the returns of the call under the physical probability measure, which is the relevant probability for portfolio optimization.

The weight of the low-risk security in the mental account increases as the joint distribution of returns becomes fatter-tailed, and the weight of the call increases as well. The expected return declines and the higher-order moments increase. More generally, the risk constraint becomes increasingly hard to satisfy without accepting lower expected returns as the tails of the joint distribution become fatter.

6. Conclusion

Options and structured products have no place in optimal mean–variance portfolios, but they have a place in optimal behavioral portfolios. Behavioral portfolios are structured as collections of mental account sub-portfolios associated with particular goals. Risk is measured by the probability of failing to reach the threshold level of the goal associated with a mental account sub-portfolio, or by expected shortfall. Investors in behavioral portfolios display a range of risk tolerances, each associated with a particular goal. Thus they can be very risk averse with money in 'downside protection' mental account sub-portfolios, while they are much less risk-averse, even risk-seeking, with money in 'upside potential' mental account sub-portfolios. A complementary analysis that parallels this mental account structure may be found in de Vries (2012).

We find that particular options and structured products are suitable for particular goals. For example, put options are suitable for downside protection mental accounts associated with preventing poverty, whereas call options are optimal for upside potential mental accounts associated with a shot at riches.

Our analysis goes some way to explain the popularity of options and structured products in portfolios but it does not go far enough to explain all the features of the rich menu of financial products included in many portfolios. For example, capital guaranteed notes are structured with a floor at a nominal amount, say $10,000, paid by investors. Why do investors find such a floor attractive, given that the nominal $10,000 received 10 years later, when the note matures, implies a loss when assessed in real dollars or when relative to investment in a risk-free security? The answer likely involves consideration of loss-aversion coupled with 'money illusion,' where mental accounting is conducted in nominal dollars. Another illusion which we call 'index illusion' likely plays a role. Capital guaranteed notes often promise payments linked to increases in an index, such as the S&P 500 Index, yet not all investors are aware that indexes account for dividends and that a 100% link to an index does not imply that they receive the total return of the index.

Our setting in this paper is a static buy-and-hold setting. Examining options and structured products in dynamic settings is a useful next step. Our mental accounting framework can also be extended to the incomplete markets dynamic model[6] as well as dynamic annuitization models.[7] Finally, alternate objectives such as minimizing tracking error variance in this framework may also be pursued, see Alexander and Baptista (2008).

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Appendix A. Computational algorithm for the optimization problem

Here we present the procedure to solve the problem specified in Eq. (3).

(1) Generate the state space U. Assume that of the n securities, n' are "primary" securities, i.e., they form the initial set of securities. The remaining $(n-n')$ securities are "derivative" securities, derived from the set of n' securities. To generate the partial state space U' of primary securities, we select a supporting range of returns over the initial securities (with equal increments $dr_i, i=1,\dots,n'$), and then convolute the supports into an overall state space (i.e, obtain the product state-space). For example, if we choose m values in each security support, then the size of the state space is $m^{n'}$. Formally, $U'=\{r(u)\}, u=1,\dots,m^{n'}$. Now we can expand this state space U' into the full state space U by augmenting the space with the derivative securities which are written as functions of the initial securities. We note that U may be represented by a matrix of returns, i.e., $U \in R^{m^{n'} \times n}$. The returns of all securities in each state are given by the rows of this matrix.
(2) For each $u \in U'$, compute the probability of the state $p[r(u)] \in R^{m^{n'}}$, which is computed using the multivariate normal density function, if the joint distribution is normal, or more generally, using copulas, if the distribution is non-normal.
(3) Given all the components of the objective function in Eq. (3), find the vector of weights $w \in R^n$ that maximizes the objective, subject to the constraints in Eqs. (4) and (5).

Maximization of the objective function is a non-trivial computational task, and we develop a robust, stable and fast two-stage approach. First, we undertake a sparse grid search over a set of possible portfolio weights in order to locate the region of the global optimum. Second, we use a gradient optimizer in this region to locate (climb up to) the optimal portfolio. We provide more detail as follows.

(1) *Grid search step*: There are two ways in which we may generate a grid of possible values of the vector $w \in R^n$. One, we pick a set of values for $\{w_1, w_2, \dots, w_{n-1}\}$ and then set $w_n = 1-\sum_{i=1}^{n-1} w_i$. This involves running $(n-1)$ loops over the set of the first $(n-1)$ weights. The coarseness of the support chosen for each w_i may be varied to trade-off accuracy versus speed.[8]
For the computations in the paper, all examples comprise four assets, and a sparse grid of values of $\{w_1, w_2, w_3, w_4\} \in [0,1]$ is constructed with constant step $\Delta w = 0.05$ (i.e., 21 values each). So, w_1 is chosen in [0, 1], $w_2 \in [0, 1-w_1]$, $w_3 \in [0, 1-w_1-w_2]$,and $w_4 = 1-w_1-w_2-w_3$. Nested loops are used to set up the grid. In the optimization step, constrained optimization is used to ensure that $w \geq \mathbf{0}$.
For each possible set of portfolio weights w we compute the portfolio returns for each state u. We then examine the vector of portfolio returns to check if the constraint $\int_{u:[w' r(u)] < H} p[r(u)] \cdot \prod_{i=1}^{n} dr_i(u) \leq \alpha$ is satisfied. We discard all w sets that do not meet this condition, leaving only "eligible" sets of portfolio weights. We then compute the expected return for each eligible set of weights to find the one set that gives the highest expected return. This set delivers the starting guess for maximized weights that is entered into the optimization step of the algorithm.

[6] There is a literature on dynamic portfolio choice with VaR constraints (see Basak and Shapiro, 2001; Cuoco and Liu, 2006; Cuoco et al., 2008, for example), but none of these has examined the use of derivative securities such as the ones examined here. For incomplete markets see the dynamic model proposed in Basak and Chabakauri (2010).

[7] See Milevsky and Young (2007).

[8] As an alternative, we may simulate a grid of w values for the first $(n-1)$ securities using a low-discrepancy Sobol (1967, 1976) sequence generator, or some other method for generating low-discrepancy state spaces. These generators are known for their facility in providing good coverings of the weight-space. These sequences are also known as quasi-random or sub-random sequences. The size of the weight-space here is chosen directly as required. This approach is easy as it does not involve convoluting individual weight vectors for each security. And, once again, we set $w_n = 1-\sum_{i=1}^{n-1} w_i$.

(2) *Optimization step*: The optimization problem in Eqs. (3)–(5) takes a form that is not amenable to standard optimizers. Note that we are maximizing a *nonlinear* numerical function subject to a constraint on the value of another *nonlinear* numerical function (the probability α constraint). This makes it difficult to use canned optimization packages that require either linear objective functions or constraints, or require analytic functions. However, we are able to finesse the difficulty by restating the problem as follows:

$$\begin{aligned}&\max_{w} \int_{u \in U} [w' r(u)] \cdot p[r(u)] \cdot \prod_{i=1}^{n} dr_i(u) \\ &-X\left[\alpha - \int_{u:[w'r(u)]<H} p[r(u)] \prod_{i=1}^{n} dr_i(u)\right]^2 \\ &\text{where} \quad w_n = 1 - \sum_{i=1}^{n-1} w_i \end{aligned} \tag{A.1}$$

and X is a "penalty" (scalar) value that is sufficiently large (in all examples, we use $X=1000$) so as to drive the optimizer to set the cumulative probability of failure ($\int_{u:[w'r(u)]]<H} p[r(u)] \cdot \prod_{i=1}^{n} dr_i(u)$) equal to α. Note how the constraint is now embedded in the objective function above, thereby eliminating constraint (5). By writing the last portfolio weight as a function of the other weights, we eliminate the constraint (4). Hence, we get rid of both constraints in this manner, and end up with a simple numerical function that may be handled by a nonlinear numerical optimizer. Numerical optimizers usually allow lower and upper bounds to be applied to parameters as well, which may be imposed as needed for short-sale constraints, which we assume apply in all our examples hereon. Armed with the starting solution from the grid search step, the optimization step takes only a few seconds of computing time.[9]

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[9] More details on this approach are elucidated in the excellent text by Luenberger and Ye (2010, Chapter 13), where implementing the method for varying values of X is discussed. We keep a fixed value of $X=1000$ in all our experiments, and also obtain portfolio weights and moments that are within 0.01 of the solutions in the paper for $X=\{10{,}000, 100{,}000\}$. The optimization uses the `constrOptim` solver function in `R`.

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